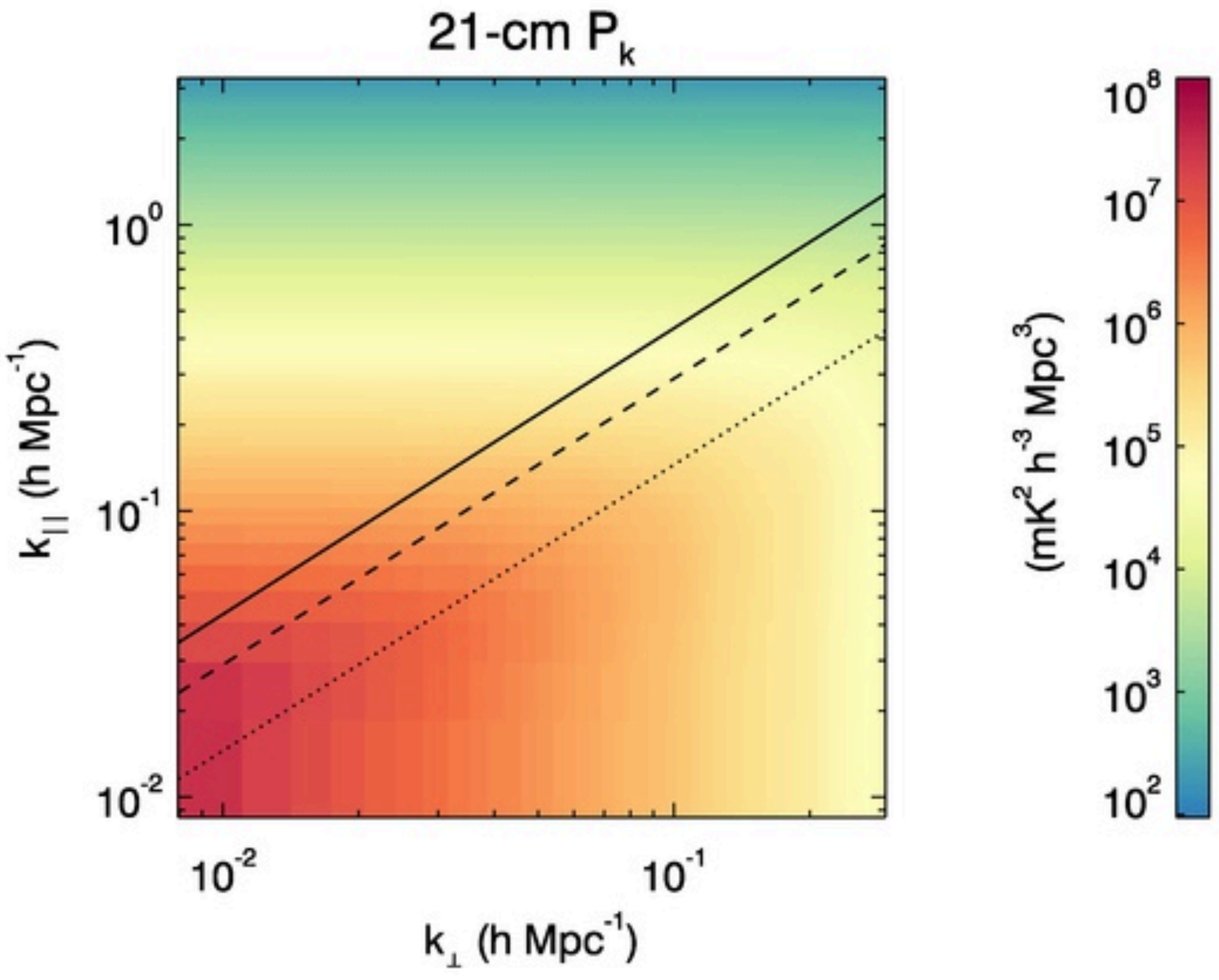
ICRAR

International Centre for Radio Astronomy Research



Estimation of the EoR Signal with an Extended MWA

Cathryn Trott
DECRA Fellow - Curtin University

CAASTRO ALL-SKY ASTROPHYSICS

Curtin University

The Government of Western Australia

The University of Western Australia

Experiment: EoR power spectrum estimation

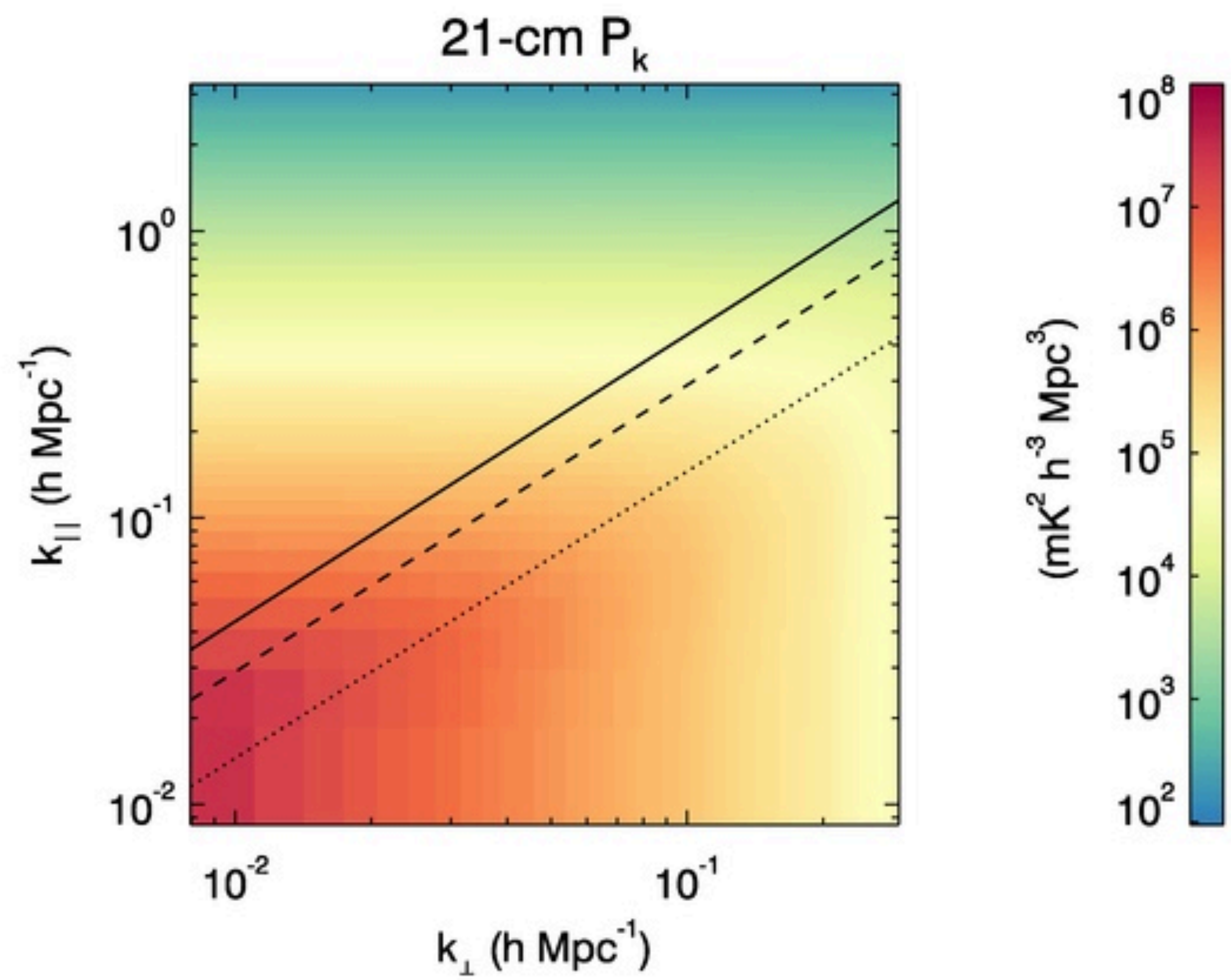
- **The spherically-averaged (1D) power spectrum measures the sky power on a given spatial scale**
  - Angular and line-of-sight considered together
  - Measures power (variance) of brightness temperature fluctuations of 21-cm HI emission line
  - Basic parameters to estimate: slope **α**, amplitude $\Delta^2(k_p)$

$$\ln\Delta^2(k) = \ln\Delta^2(k_p) + \alpha \ln\frac{k}{k_p},$$

- **The 2D power spectrum separates angular and LOS modes**
  - Crucial to discriminate foreground contaminating signals from cosmological signal

The second moment: 2D power spectral density

Line-of-sight wavenumber: spatial power

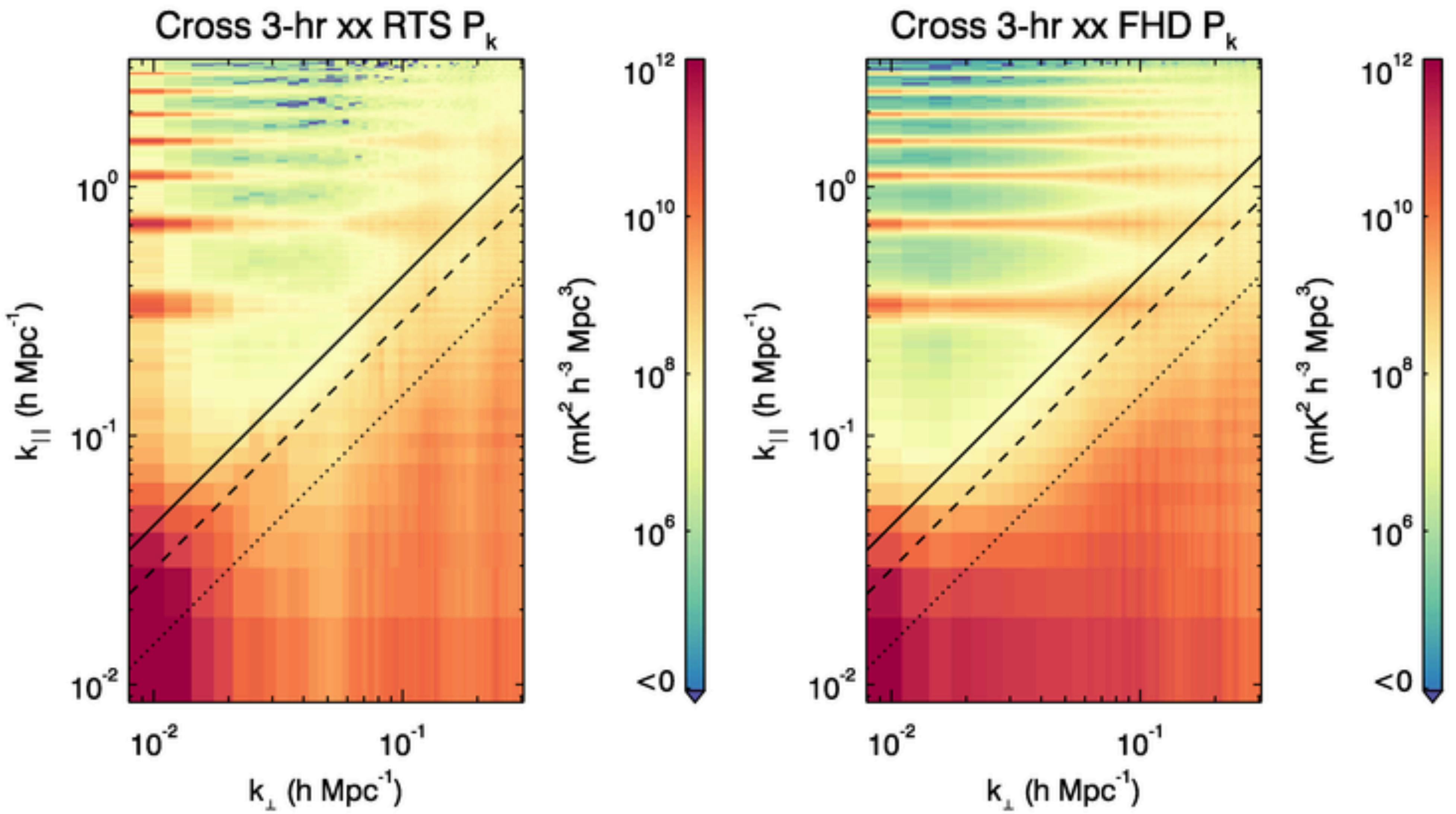


Angular wavenumber: spatial power →

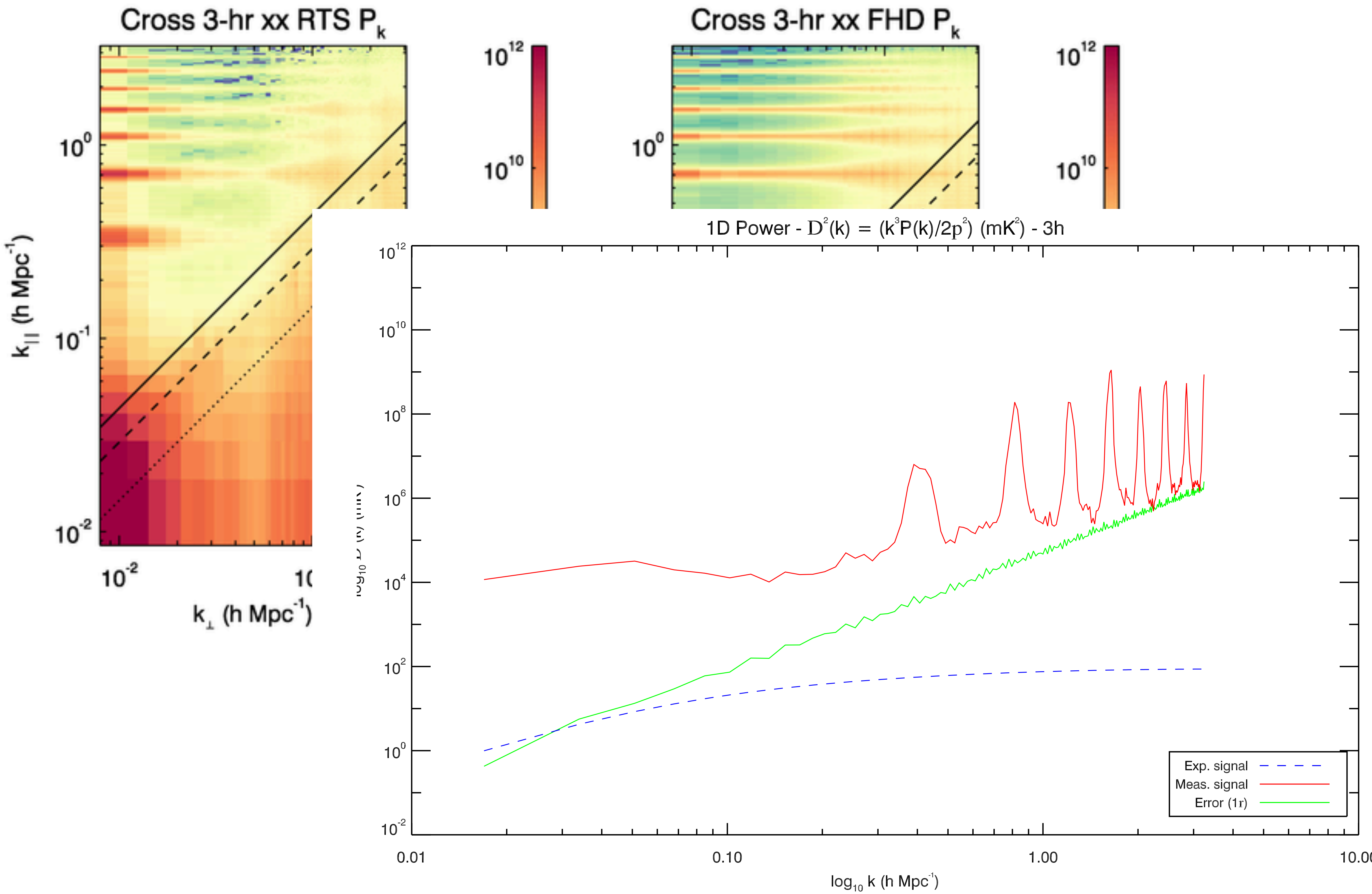
$$\langle\widehat{T}_b(\mathbf{k})^*\widehat{T}_b(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')P_T(k_\perp, k_{||}),$$

[K^2] [m^{-3}] [$m^3.K^2$]

Power spectra - examples



Power spectra - examples



Optimal estimator - experiment

Estimate of 21-cm signal, in presence of foreground signals

- Uncertainty of P_{21}: slope, amplitude: σ_α, $\sigma_{\Delta 2(kp)}$
- foregrounds --> PSF determines classical confusion level --> how deeply foregrounds can be subtracted

Optimal estimator - experiment

Estimate of 21-cm signal, in presence of foreground signals

- Uncertainty of P_{21}: slope, amplitude: σ_α, $\sigma_{\Delta 2(kp)}$
- foregrounds --> PSF determines classical confusion level --> how deeply foregrounds can be subtracted

Statistical foreground bias - **PSF-dependent**

$$\hat{P}_{21} \simeq \left(\mathcal{H}^\dagger C^{-1}\mathcal{H}\right)^{-2}\left(\mathcal{H}^\dagger C^{-1}\vec{S}_\nu\vec{S}_\nu^\dagger C^{-1}\mathcal{H}\right) - \left(\mathcal{H}^\dagger C_{\rm FG}^{-1}\mathcal{H}\right)^{-1}$$

$$\sim \chi_1^2\left(\left(\mathcal{H}^\dagger C_{21}^{-1}\mathcal{H}\right)^{-1}, 2\left(\mathcal{H}^\dagger C^{-1}\mathcal{H}\right)^{-2}\right)$$

Variance of estimate - **array layout and PSF dependent**

Optimal estimator - experiment

Estimate of 21-cm signal, in presence of foreground signals

- Uncertainty of P_{21}: slope, amplitude: σ_α, $\sigma_{\Delta 2(kp)}$
- foregrounds --> PSF determines classical confusion level --> how deeply foregrounds can be subtracted

Statistical foreground bias - **PSF-dependent**

$$\hat{P}_{21} \simeq \left(\mathcal{H}^\dagger C^{-1} \mathcal{H}\right)^{-2} \left(\mathcal{H}^\dagger C^{-1} \vec{S}_\nu \vec{S}_\nu^\dagger C^{-1} \mathcal{H}\right) - \left(\mathcal{H}^\dagger C_{\rm FG}^{-1} \mathcal{H}\right)^{-1}$$

$$\sim \chi_1^2 \left(\left(\mathcal{H}^\dagger C_{21}^{-1} \mathcal{H}\right)^{-1}, 2\left(\mathcal{H}^\dagger C^{-1} \mathcal{H}\right)^{-2} \right)$$

Variance of estimate - **array layout and PSF dependent**

Generalized data covariance

$$C(\nu) \equiv \langle S^\dagger(\nu) S(\nu) \rangle = C_{\rm FG}(\nu) + N(\nu) + C_{21}(\nu)$$

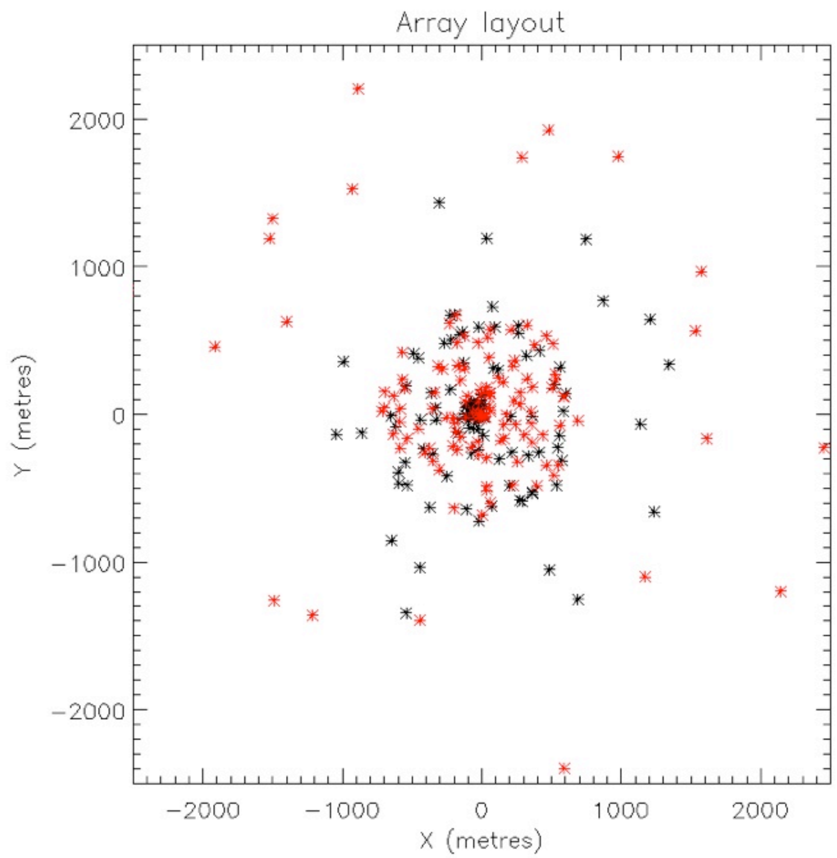
$$C(k) \equiv \langle S^\dagger(k) S(k) \rangle = C_{\rm FG}(k) + N(k) + P_{21}$$

Arrays considered

1

256TArms

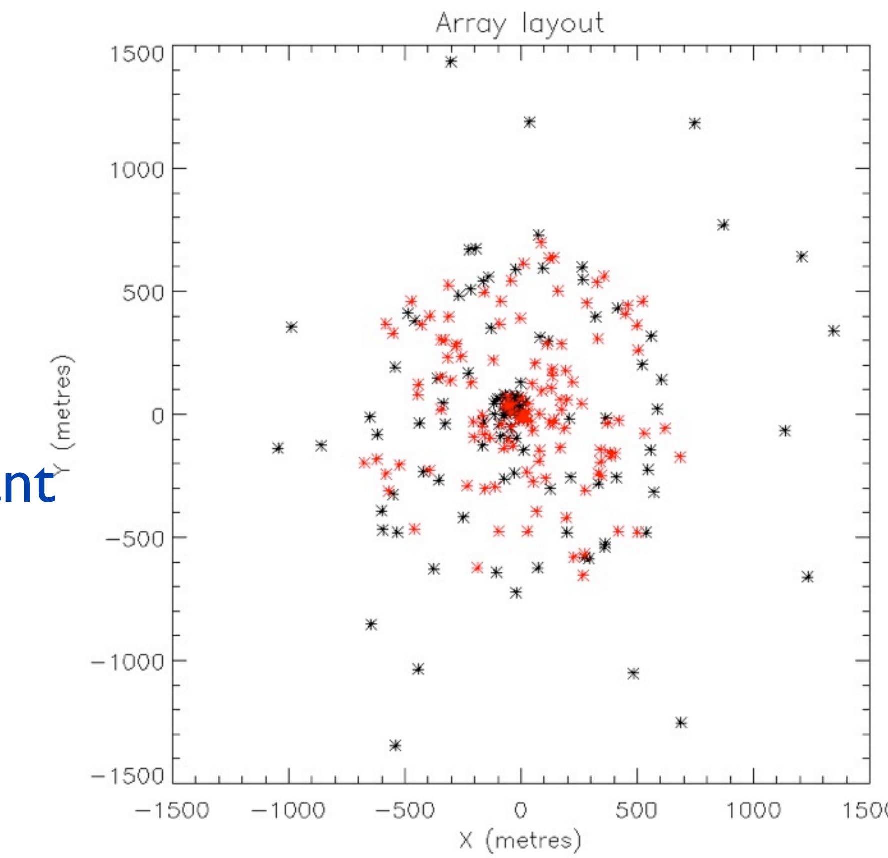
- Long baselines
- Pack core
- Non-redundant



2

256TCore

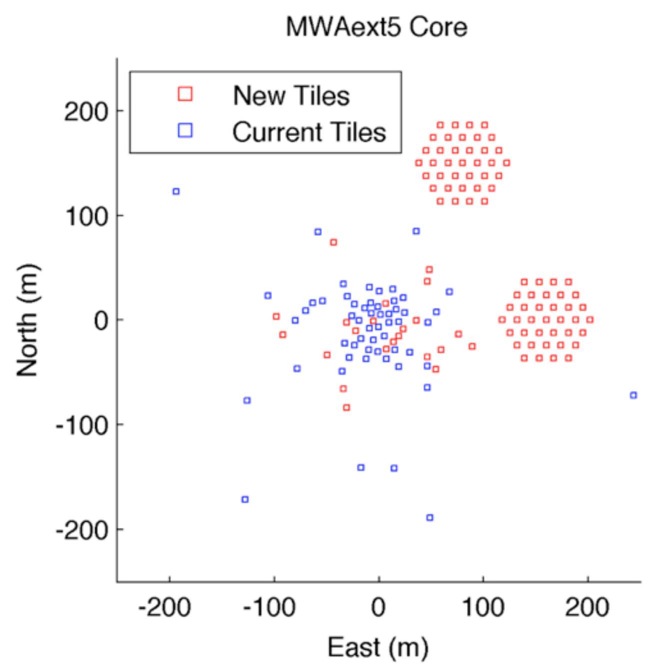
- Pack core
- Non-redundant

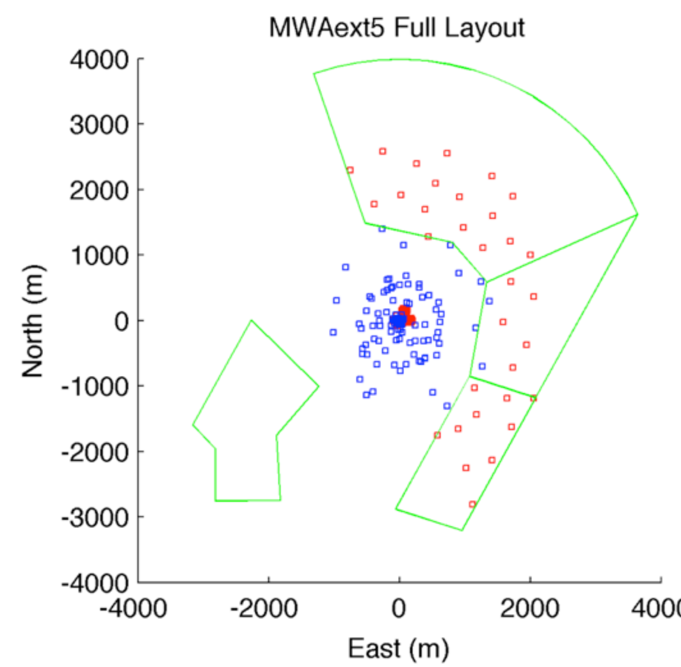


3

Hexagonal

- (Beardsley)
- Long baselines
- Redundant hexagons
- Perturbation (tile positions moved with σ=3m)





4

Current

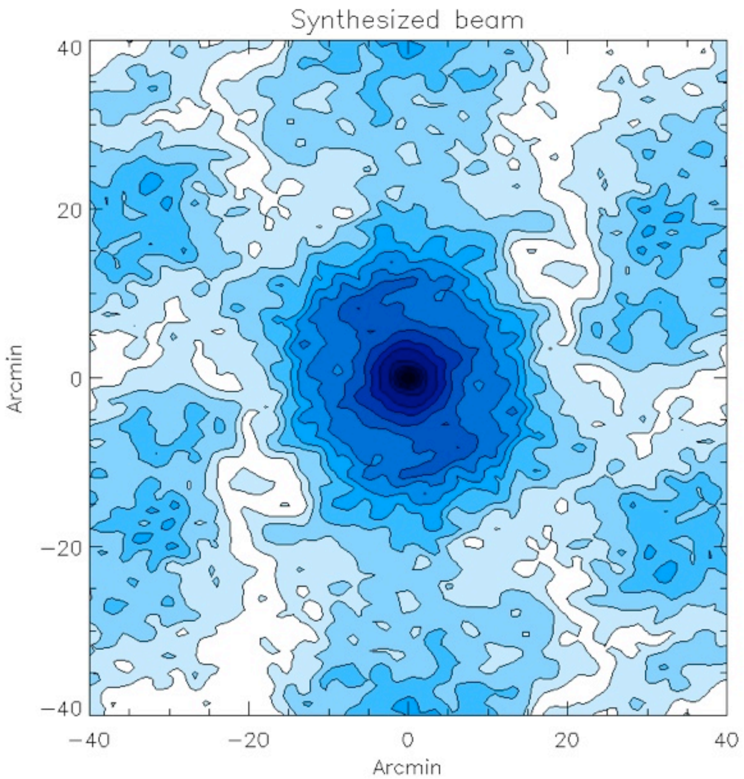
5

Smooth BP

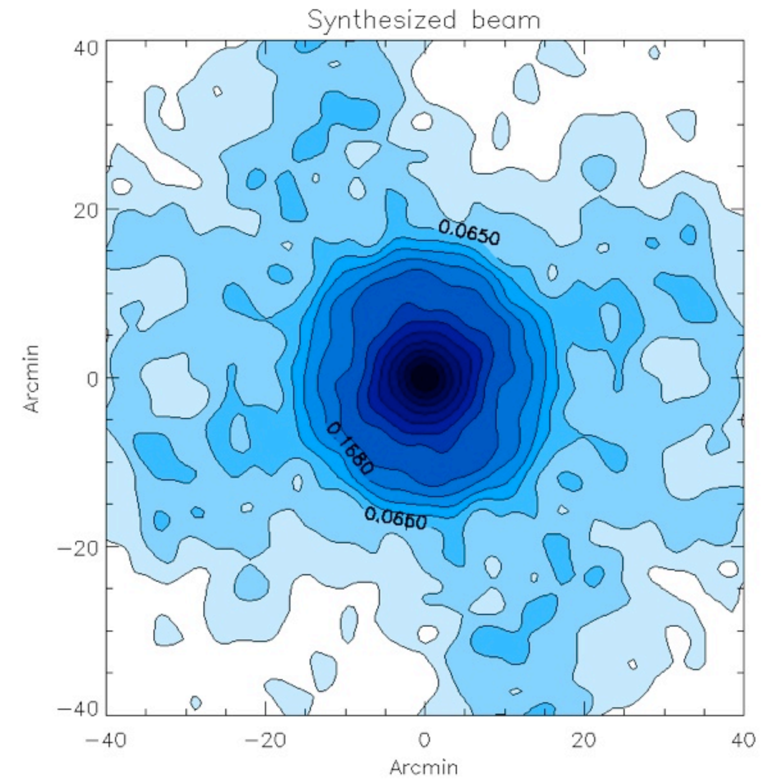
- Current instrument
- No coarse channel gaps

PSFs

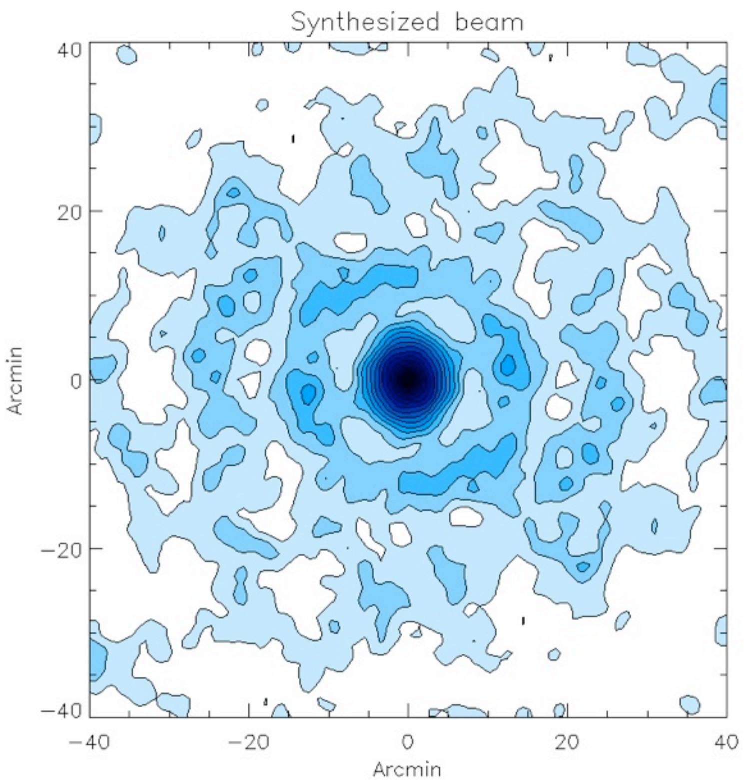
Hexagonal



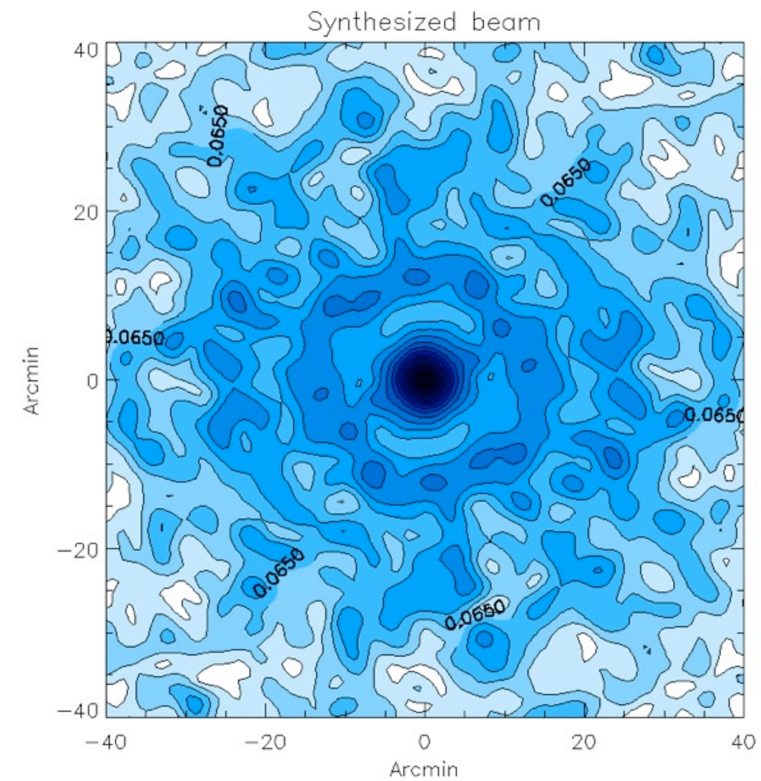
256TCore



256TArms



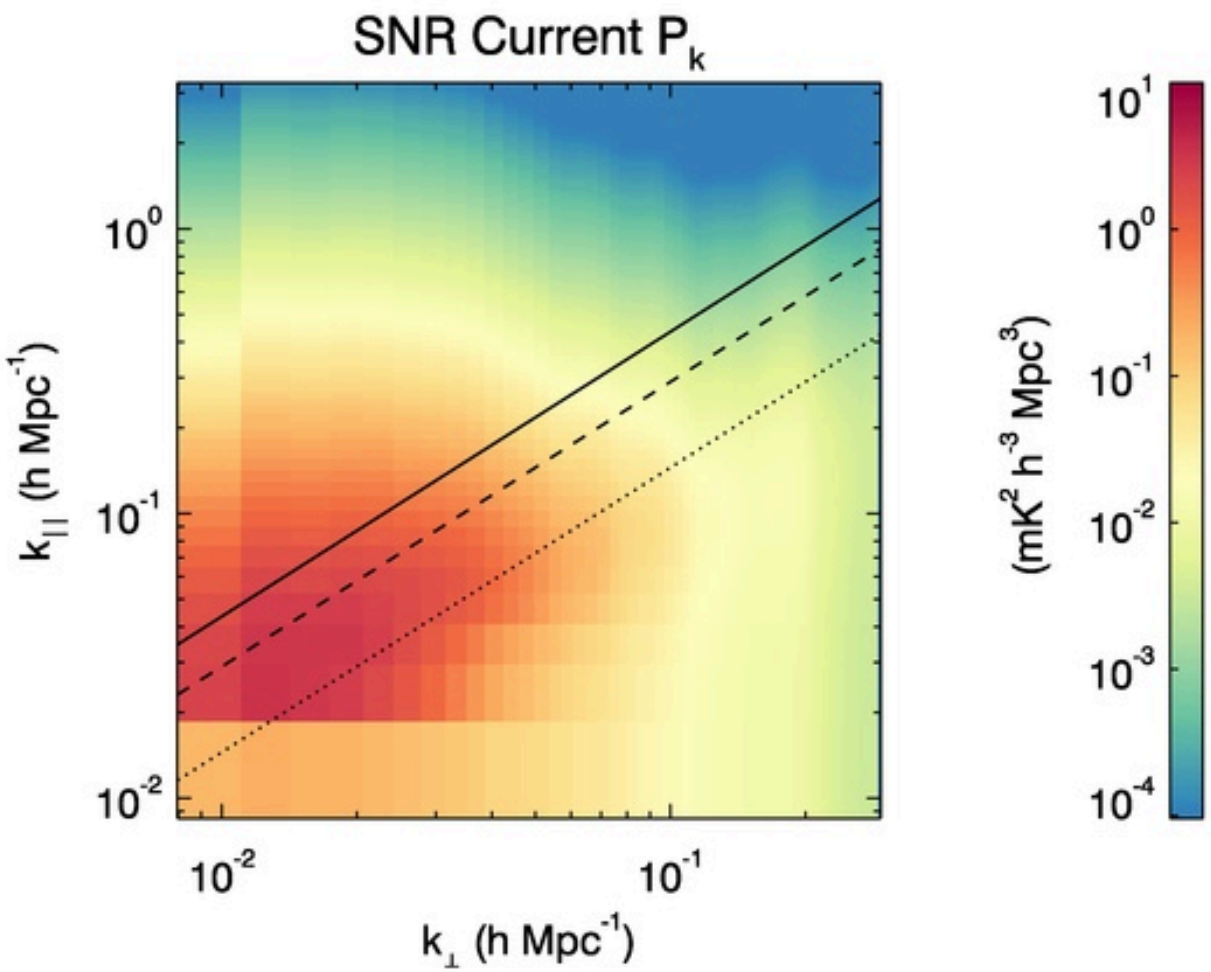
Current



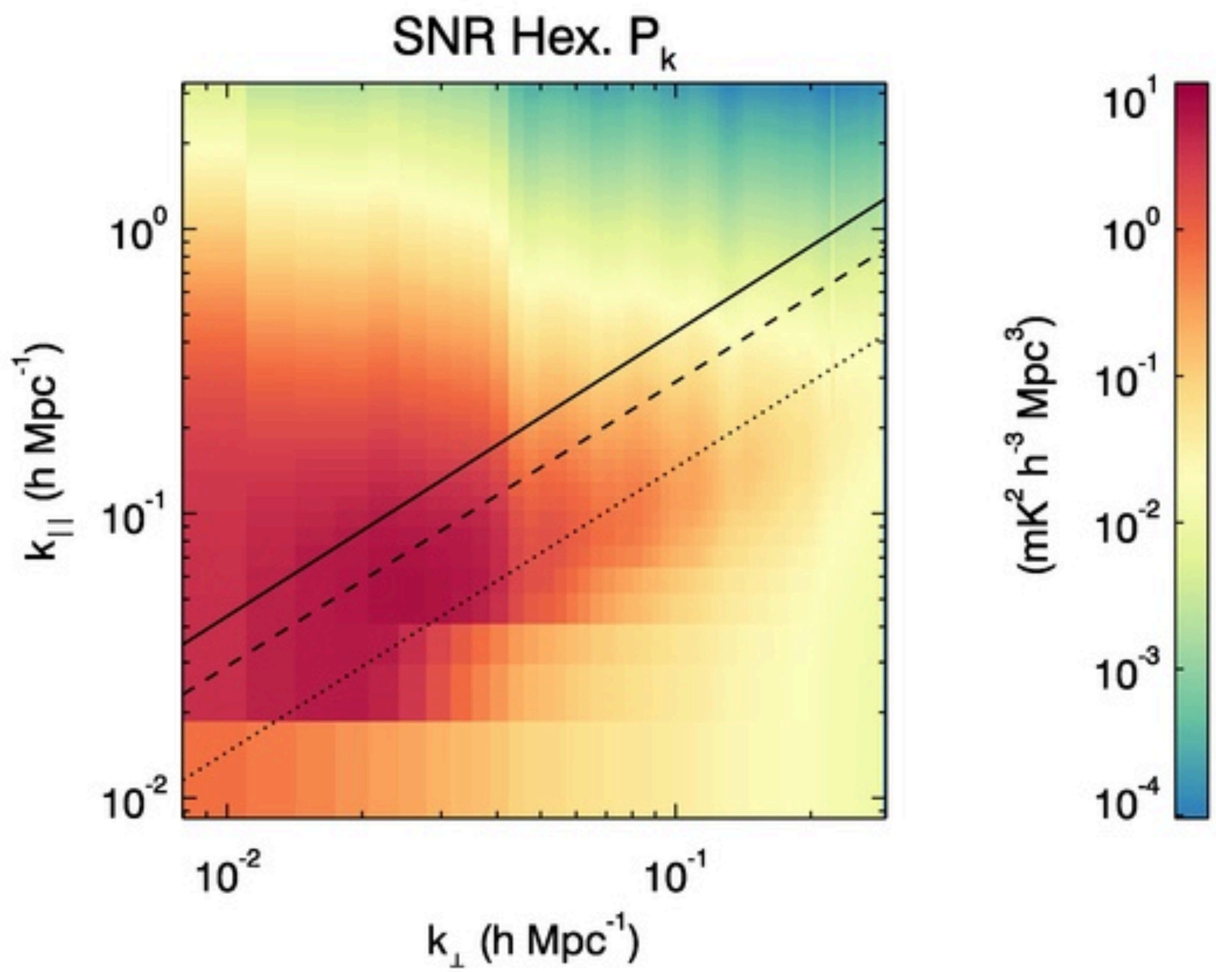
2D signal-to-noise ratios

1000 hours, 3 hour track/day

Current array | Hexagonal



Max. SNR = 3.3

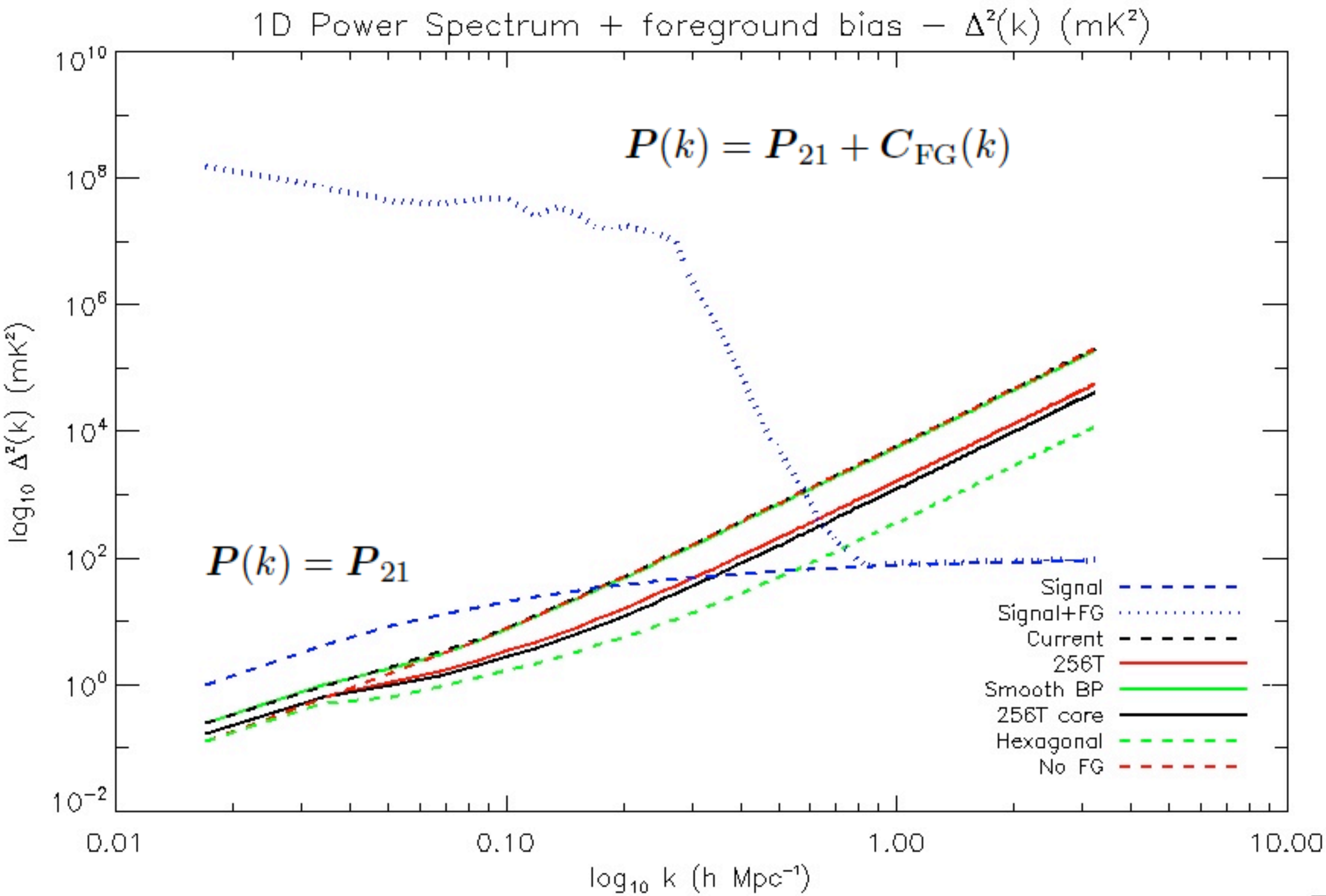


Max. SNR = 6.7

Potential for MWA+ to estimate parameters in 2D space

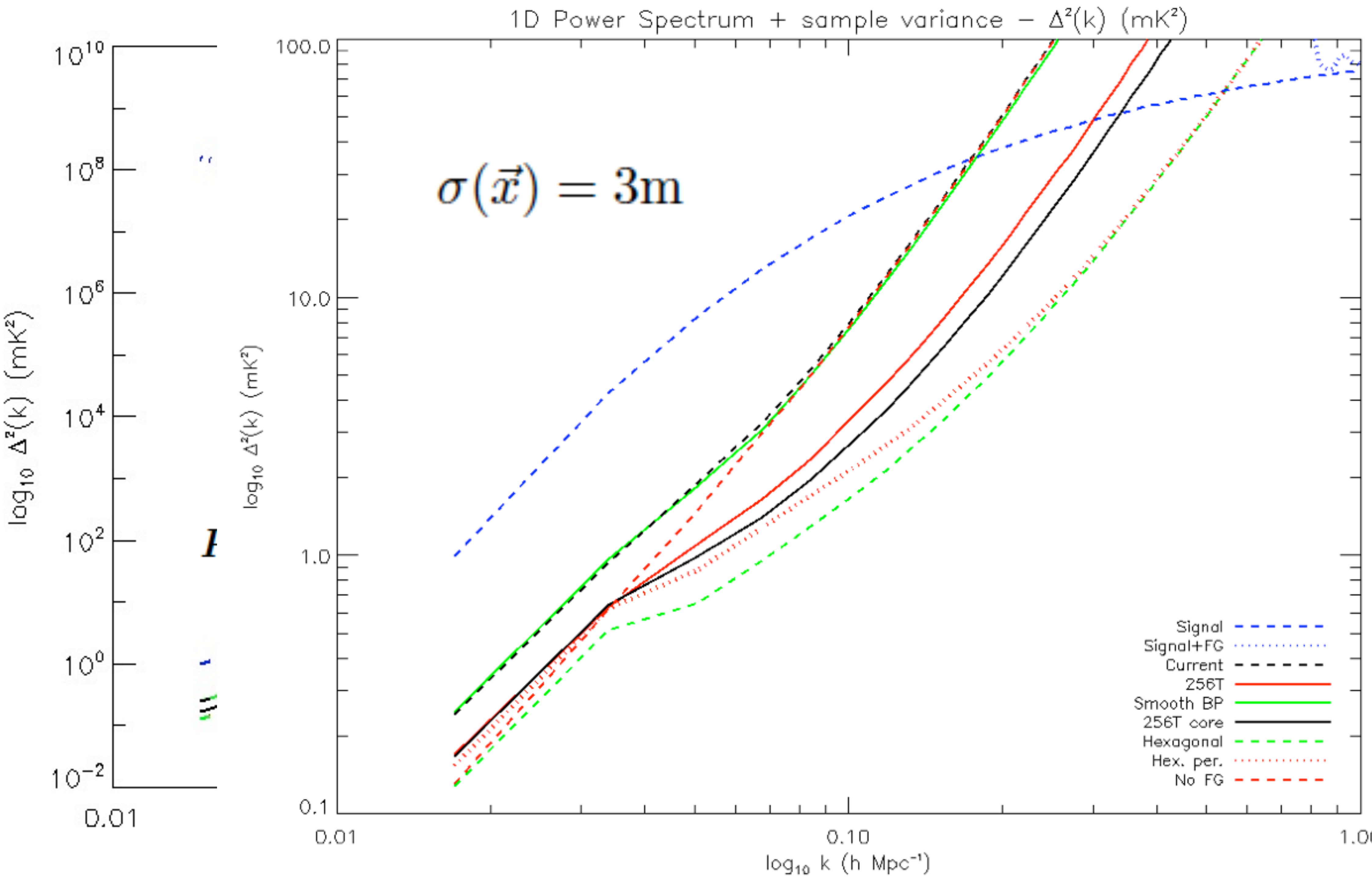
1D Power Spectra - 1000 hour experiment

$$\Delta P = \left(\sum_k \left(\mathcal{H}^\dagger C^{-1} \mathcal{H} \right)^2 \right)^{-1/2} \simeq \frac{1}{\sqrt{M_k}} (\boldsymbol{N}(k) + \boldsymbol{C}_{\mathrm{FG}}(k) + \boldsymbol{P}_{21})$$



1D Power Spectra - 1000 hour experiment

$$\Delta P = \left(\sum_k \left(\mathcal{H}^\dagger C^{-1} \mathcal{H} \right)^2 \right)^{-1/2} \simeq \frac{1}{\sqrt{M_k}} (\boldsymbol{N}(k) + \boldsymbol{C}_{\mathrm{FG}}(k) + \boldsymbol{P}_{21})$$



1D Power Spectra - SNRs - FG bias removed

$$\Delta P = \left(\sum_k \left(\mathcal{H}^\dagger C^{-1} \mathcal{H} \right)^2 \right)^{-1/2} \simeq \frac{1}{\sqrt{M_k}} (\boldsymbol{N}(k) + \boldsymbol{C}_{\mathrm{FG}}(k) + \boldsymbol{P}_{21})$$

	α/σ_α	$\Delta_p{}^2/\sigma_{\Delta p2}$
Current	8.3	6.8
256TCore	26.9	15.6
256TArms	21.3	13.2
Smooth BP	8.7	7.1
Hexagonal	55.6	25.2
Hexagonal (perturbed)	51.9	22.5

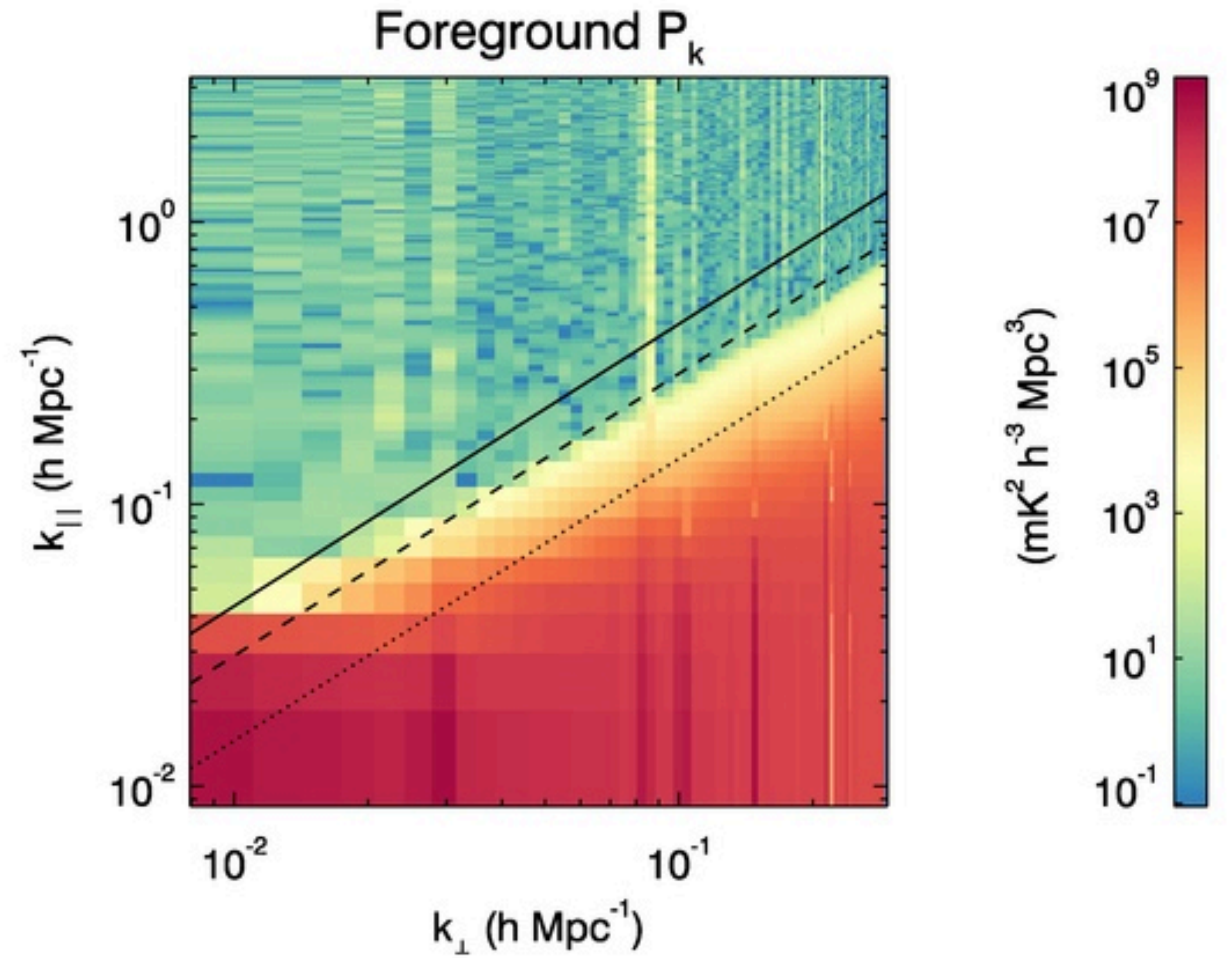
Nominal sensitivity gain, but importance is in *calibration*

Redundancy not crucial if sensitivity in right k-modes. Benefit in *calibration of systematics*

Foreground power bias

Results shown all assume the *statistical* foreground has been subtracted accurately

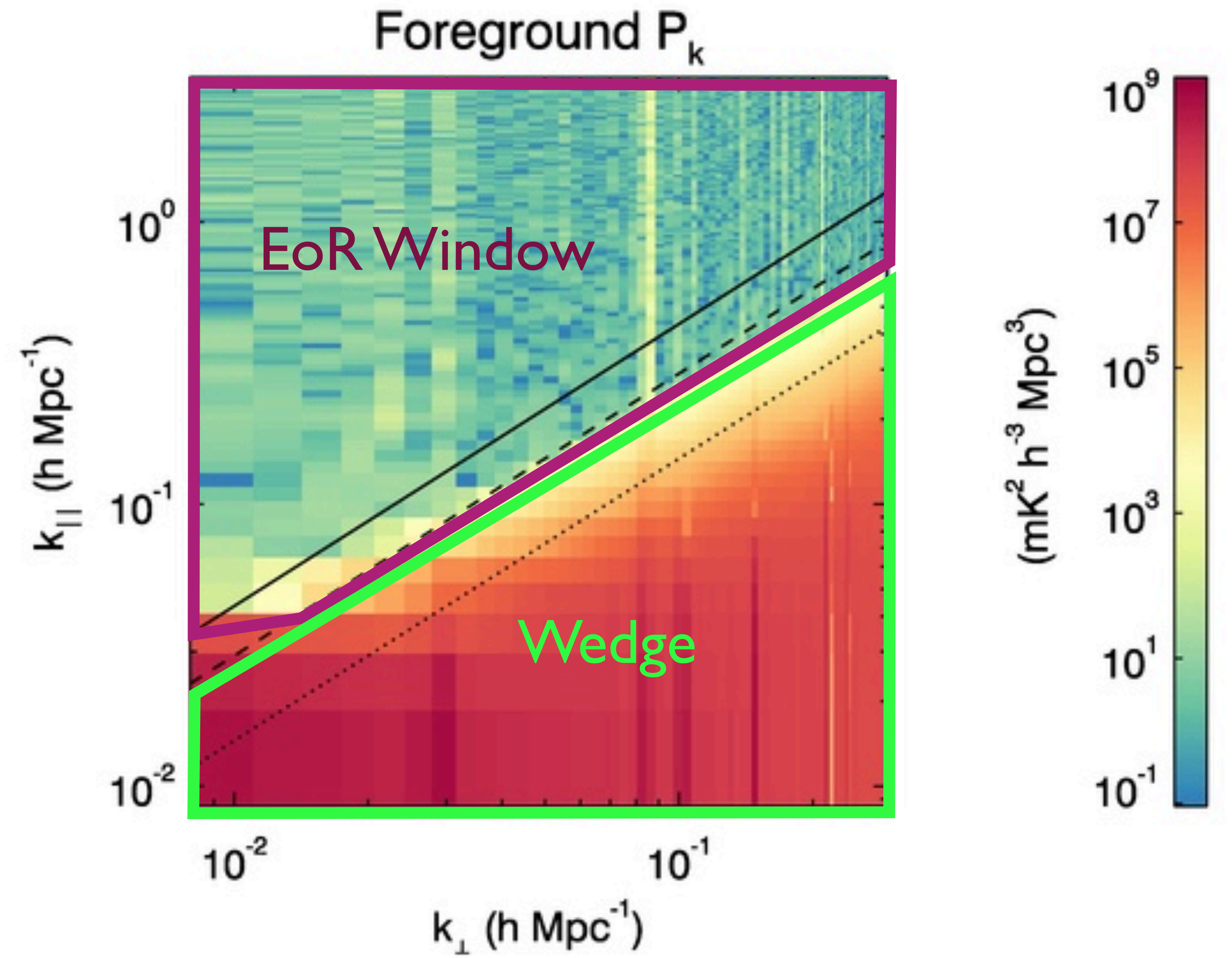
$$P(k) = P_{21}$$



Foreground power bias

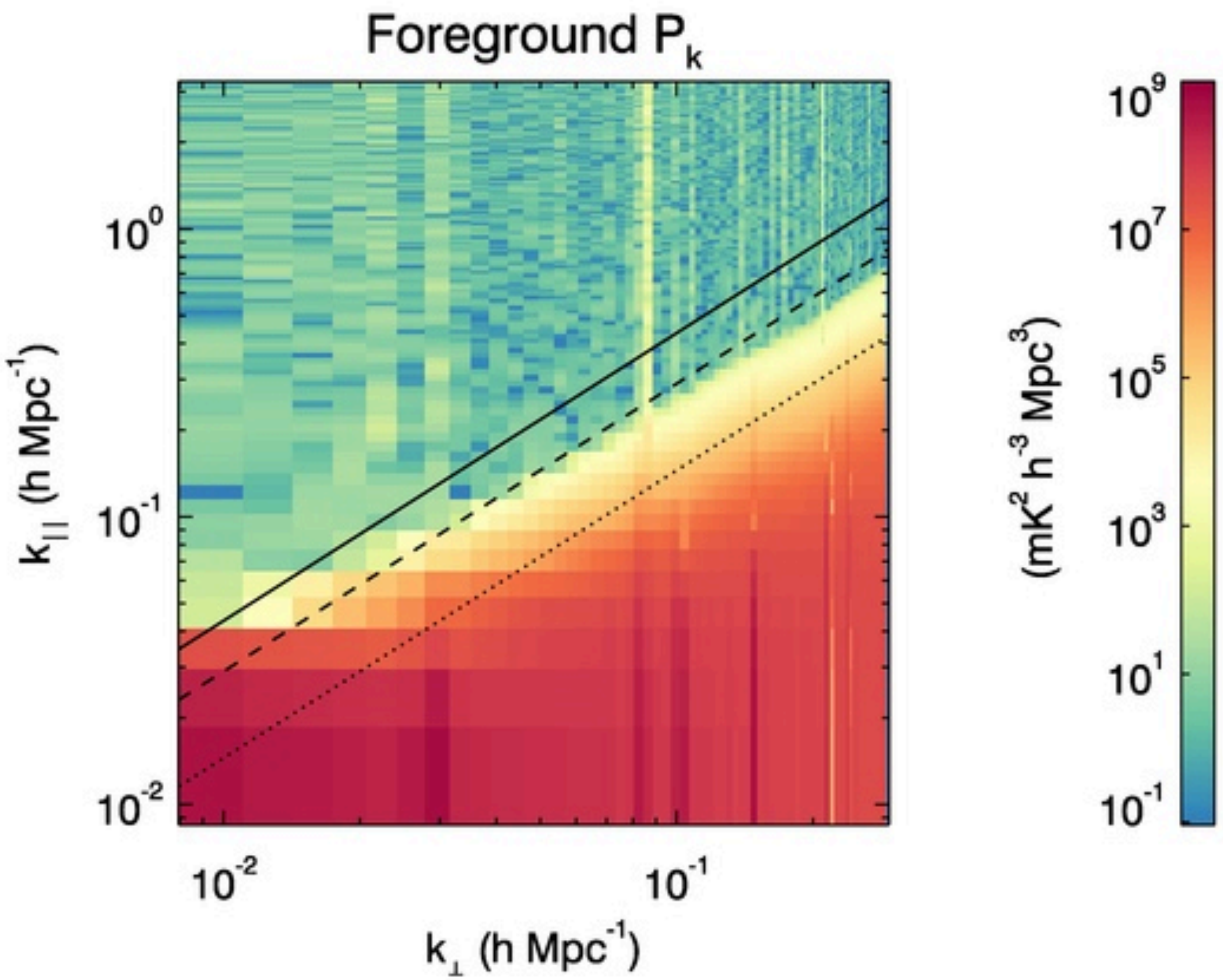
Results shown all assume the *statistical* foreground has been subtracted accurately

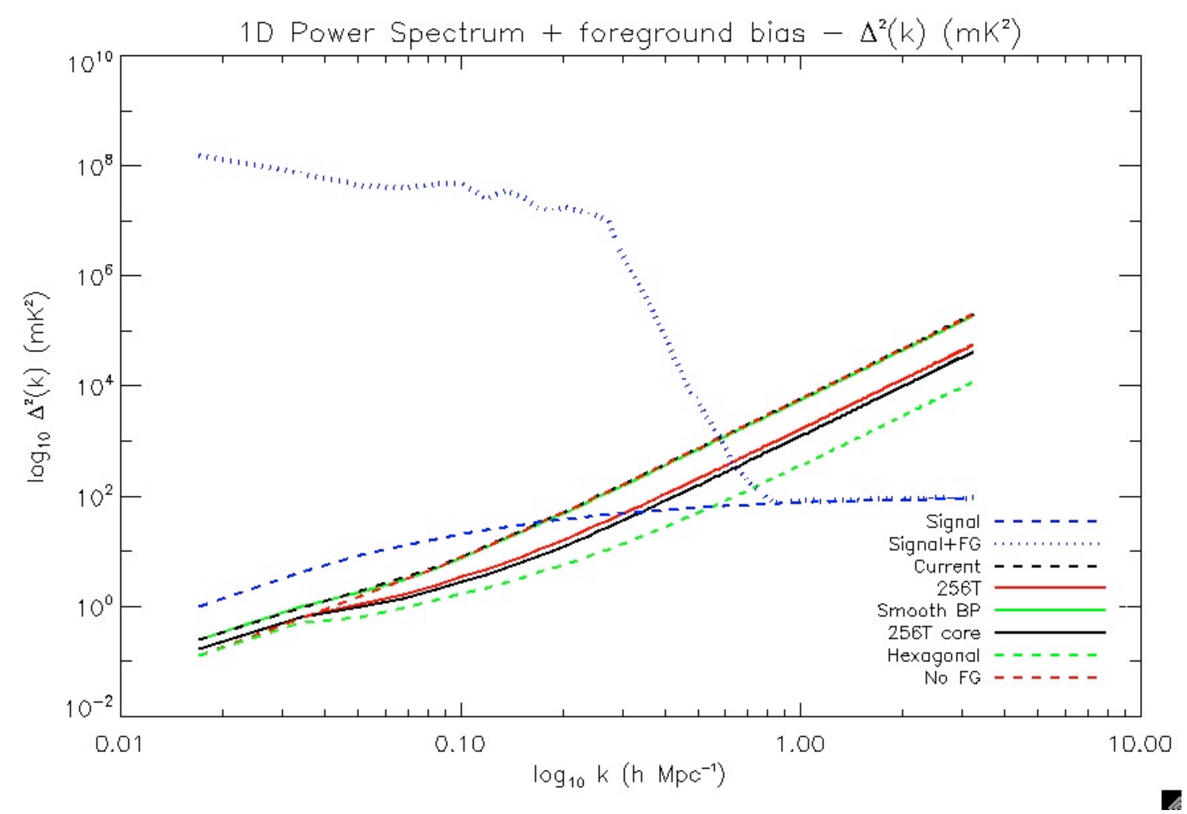
$$P(k) = P_{21}$$



Foreground power bias

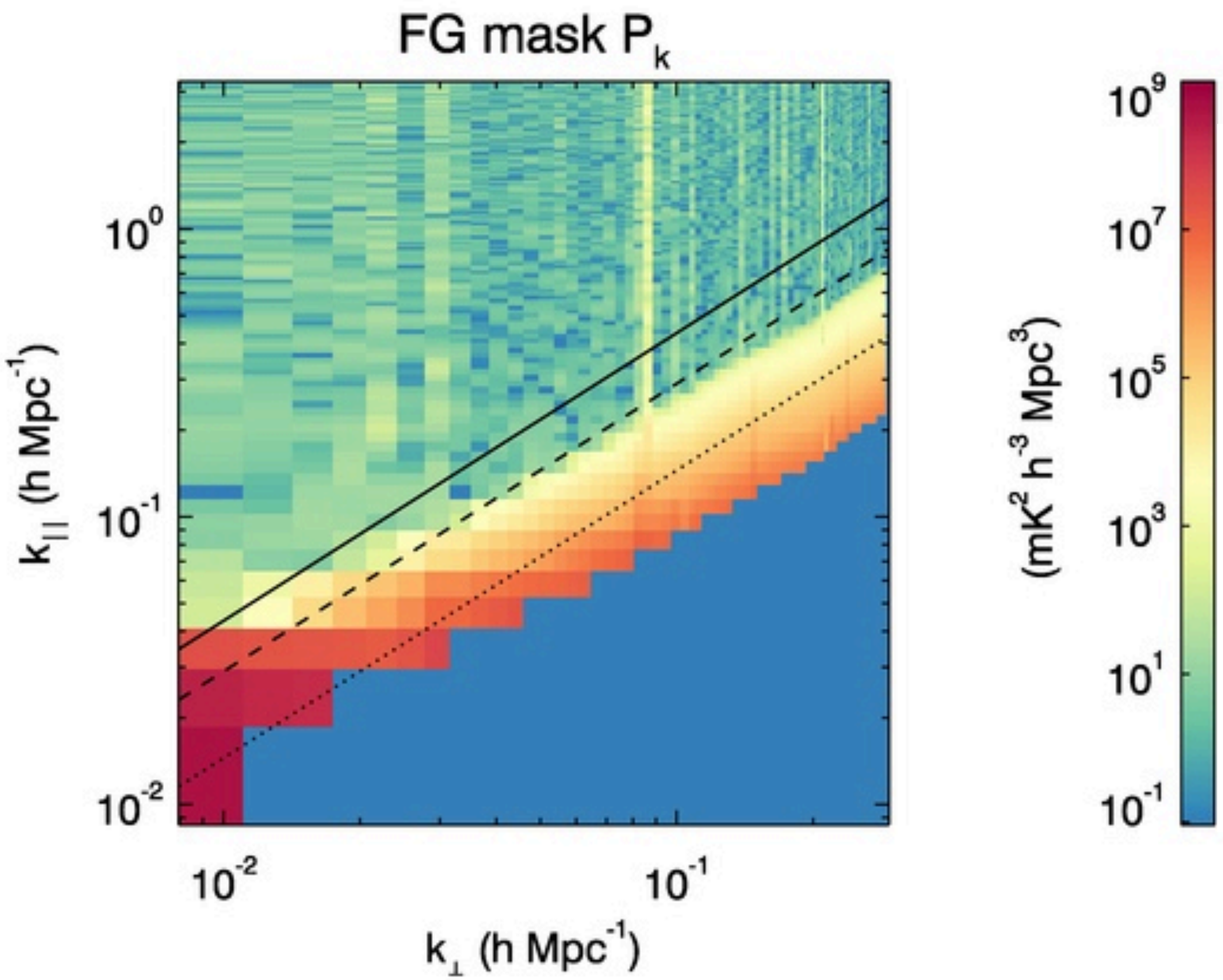
1D Power Spectrum from all data

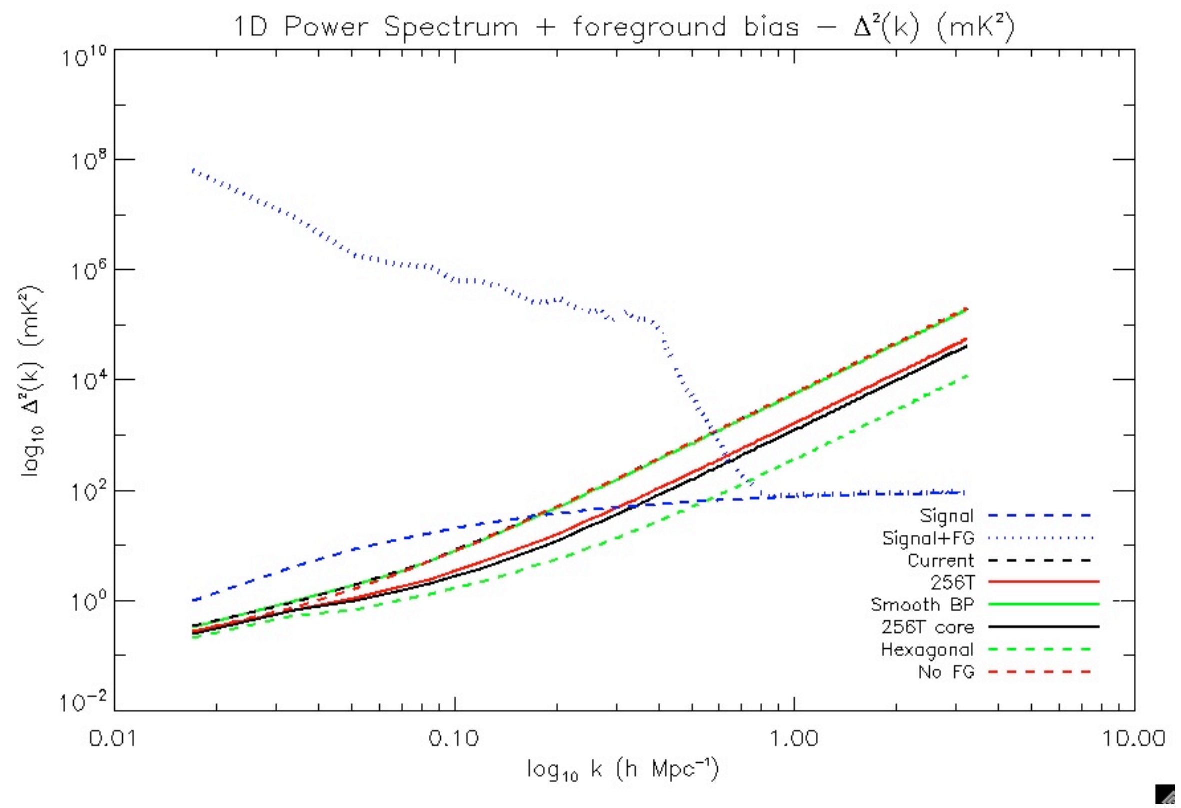




Foreground power bias

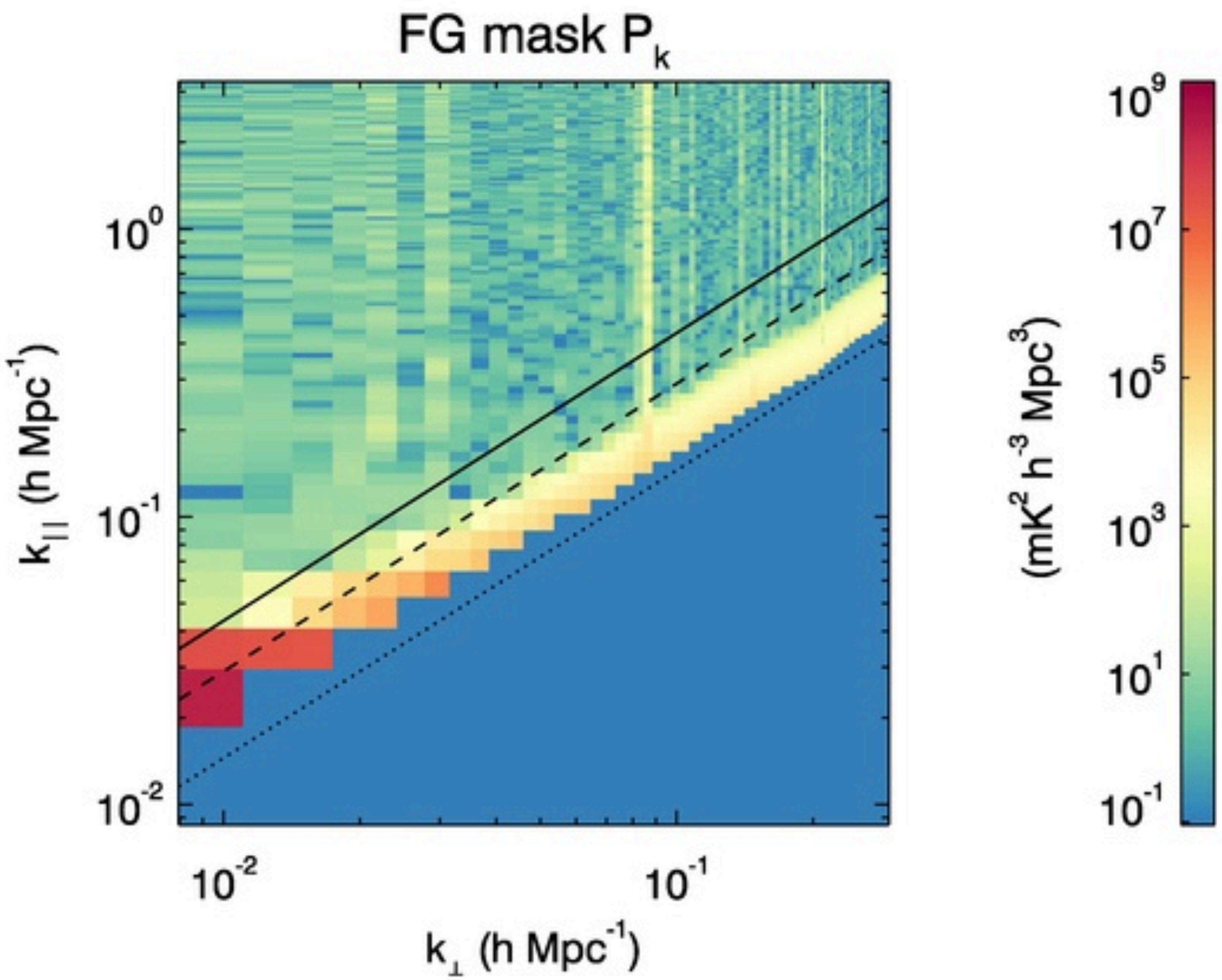
1D Power Spectrum - exclude $k_{||} > k_{\perp}$

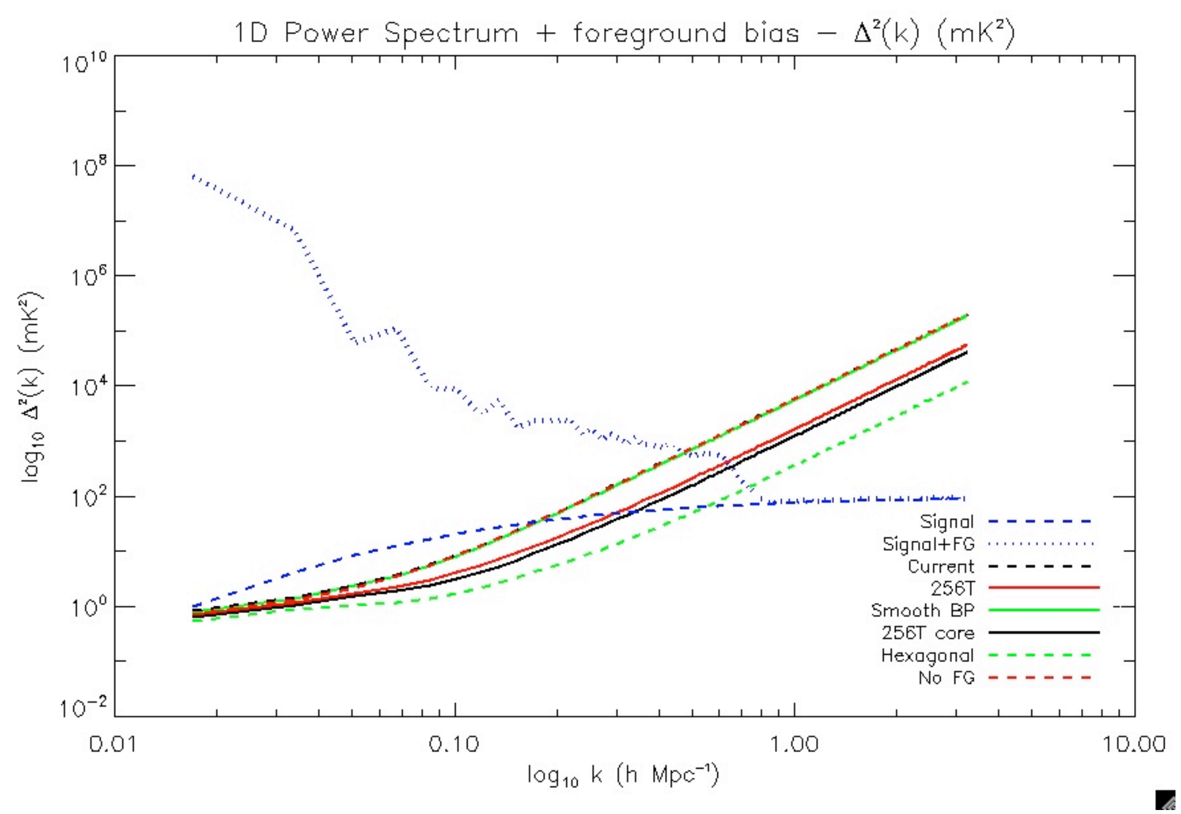




Foreground power bias

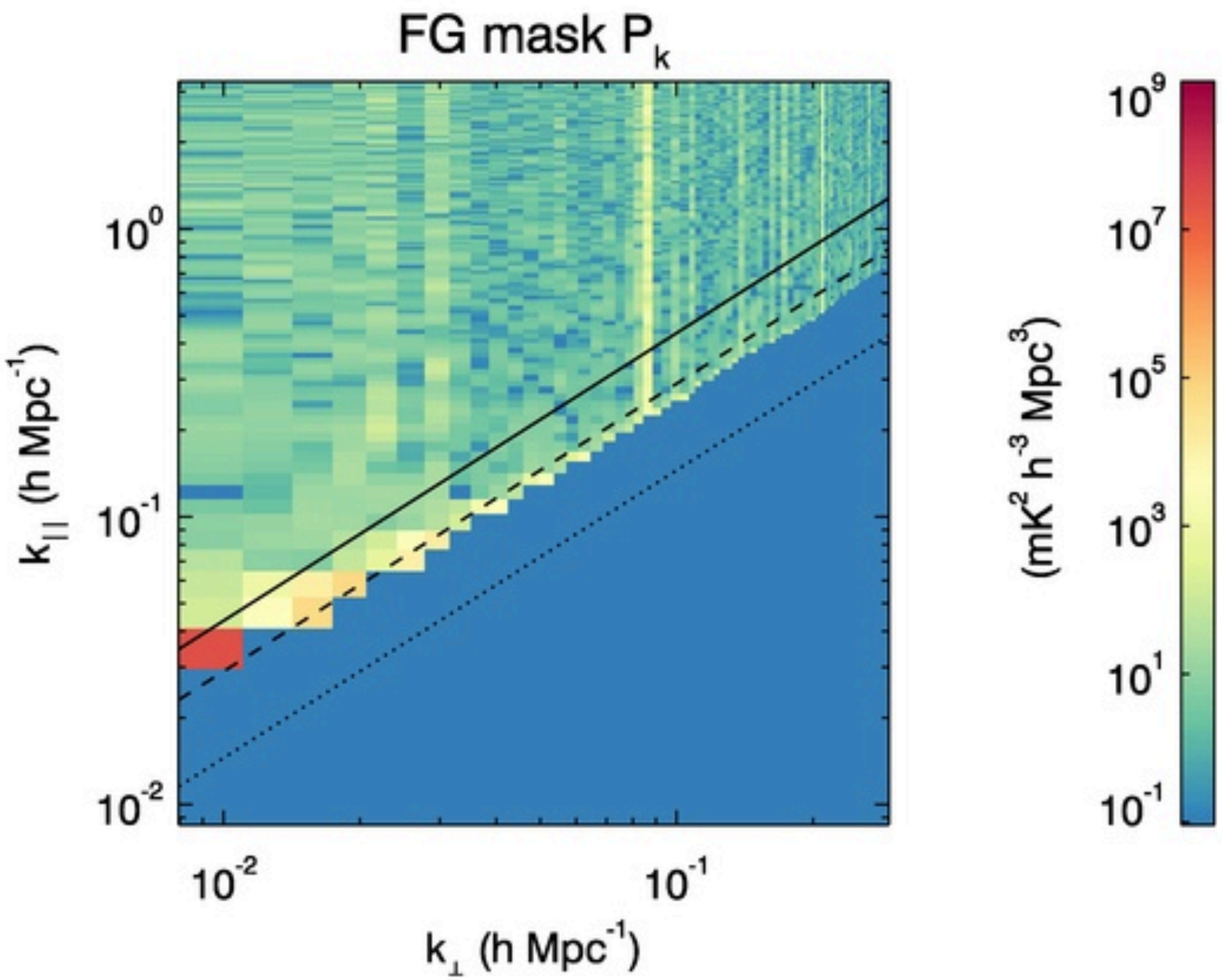
1D Power Spectrum - exclude $k_{||} > 2k_{\perp}$

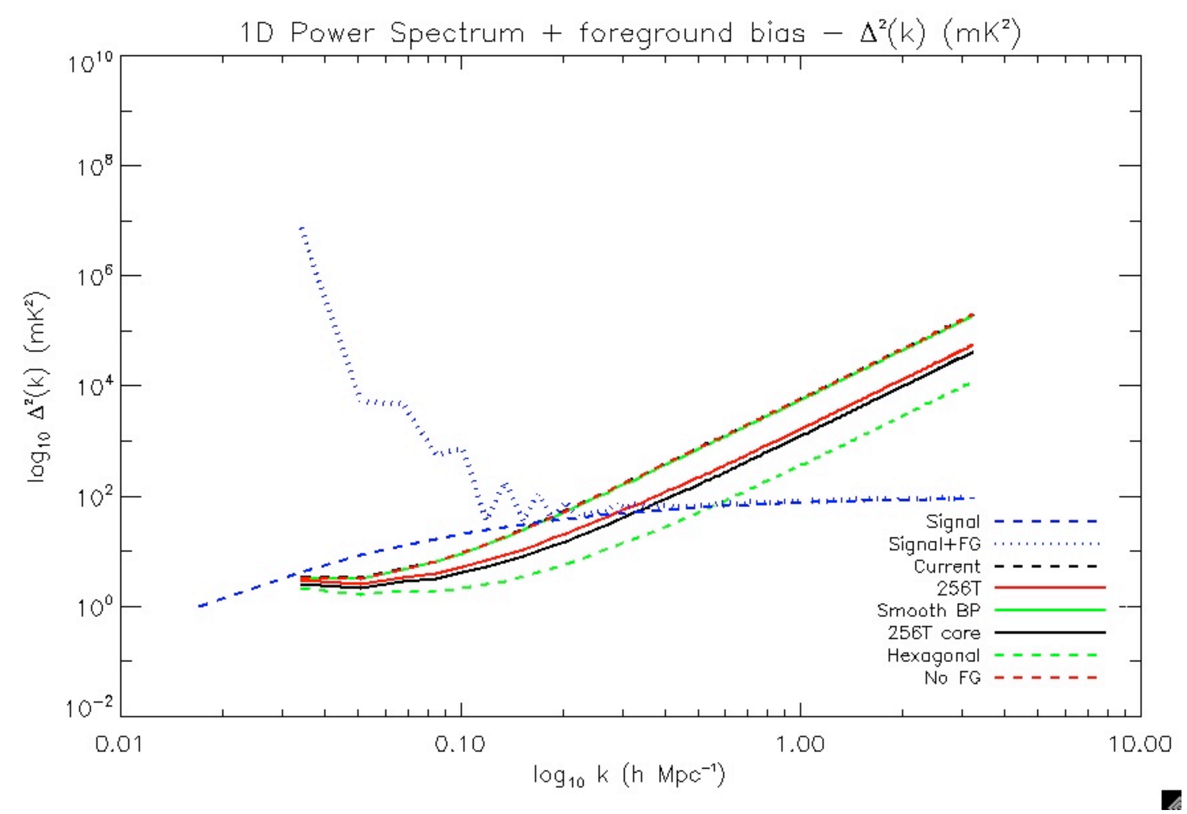




Foreground power bias

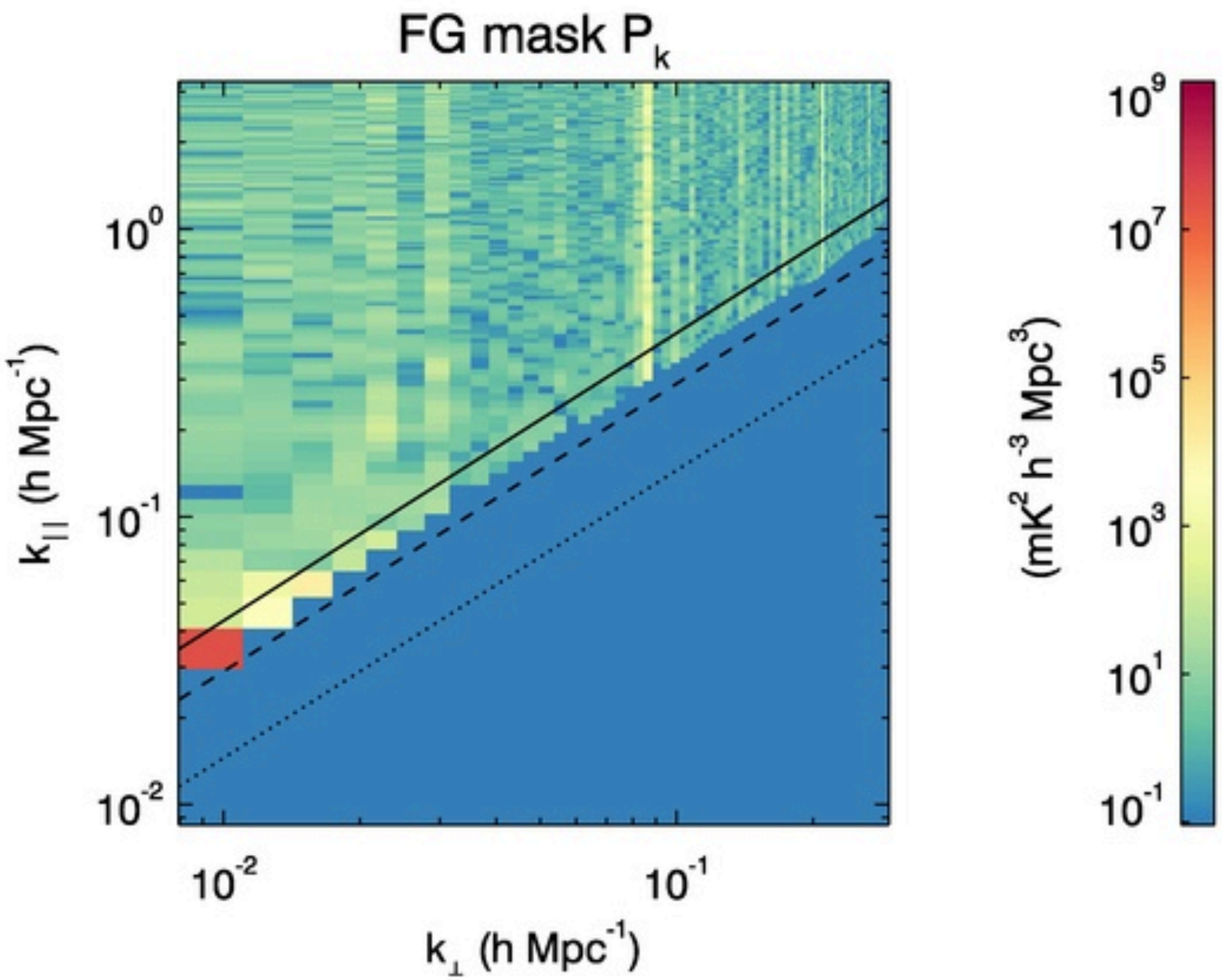
1D Power Spectrum - exclude $k_{||} > 3k_{\perp}$

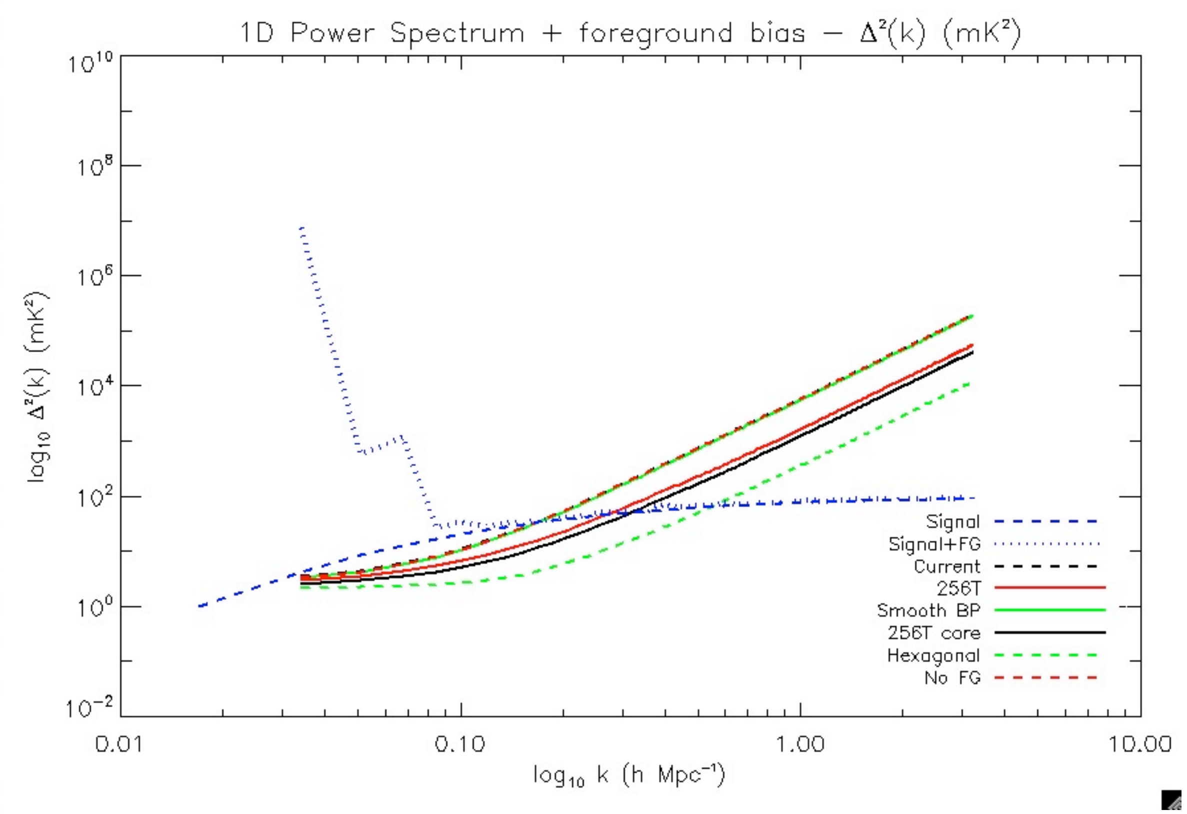




Foreground power bias

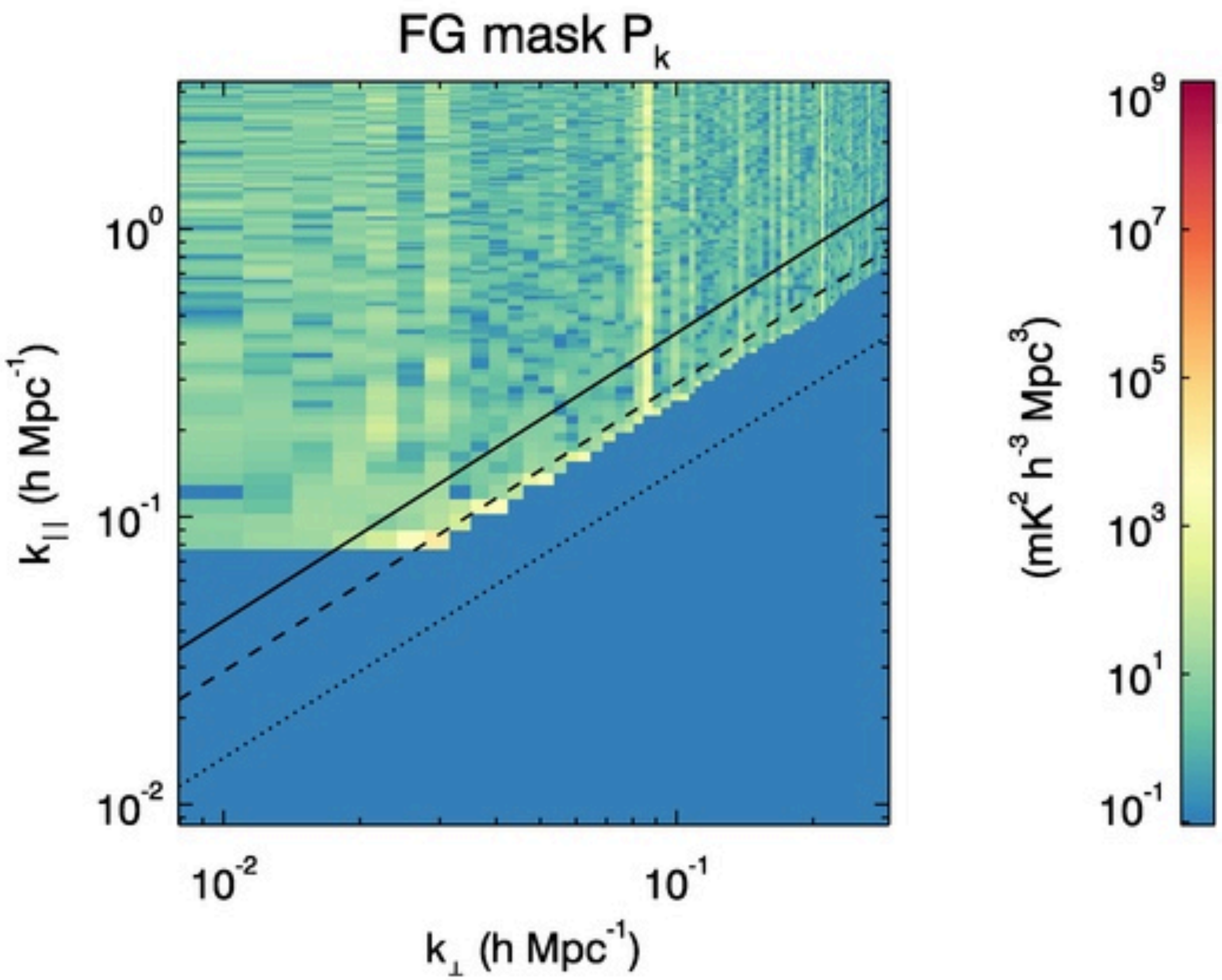
1D Power Spectrum - exclude $k_{||} > 4k_{\perp}$

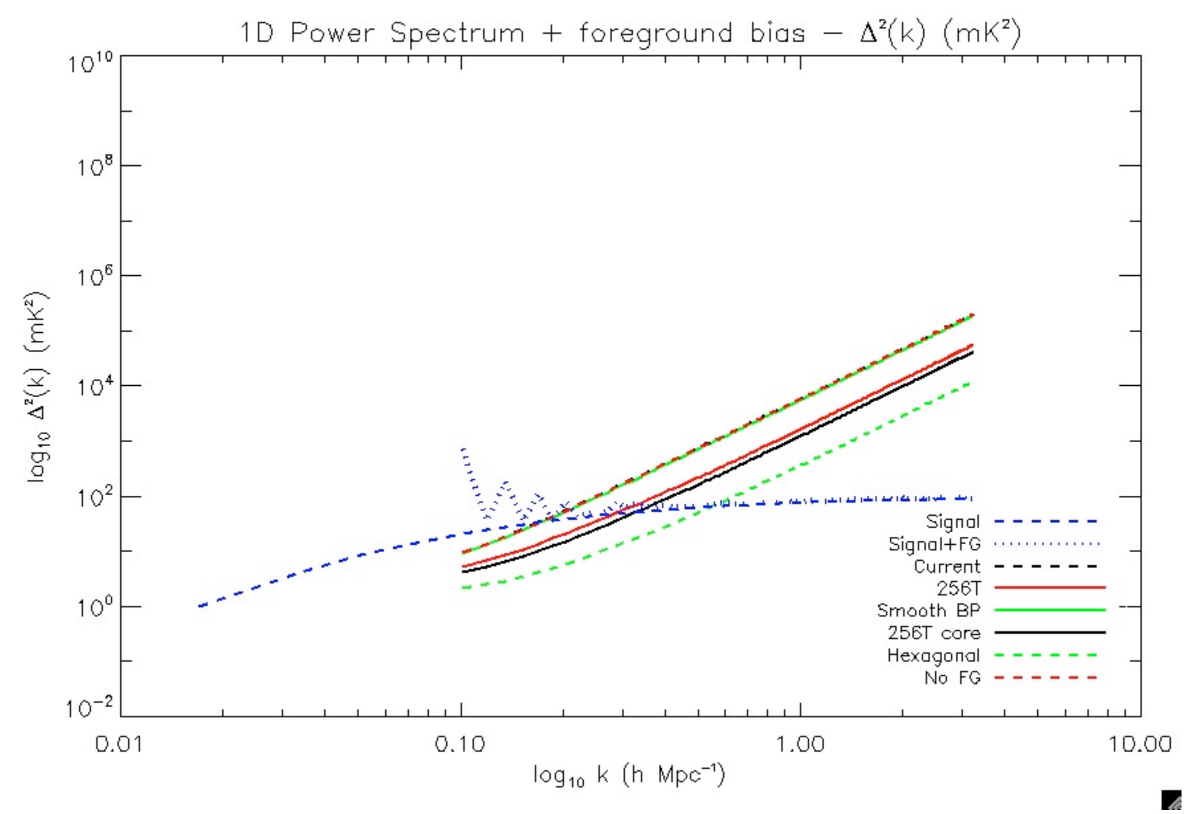




Foreground power bias

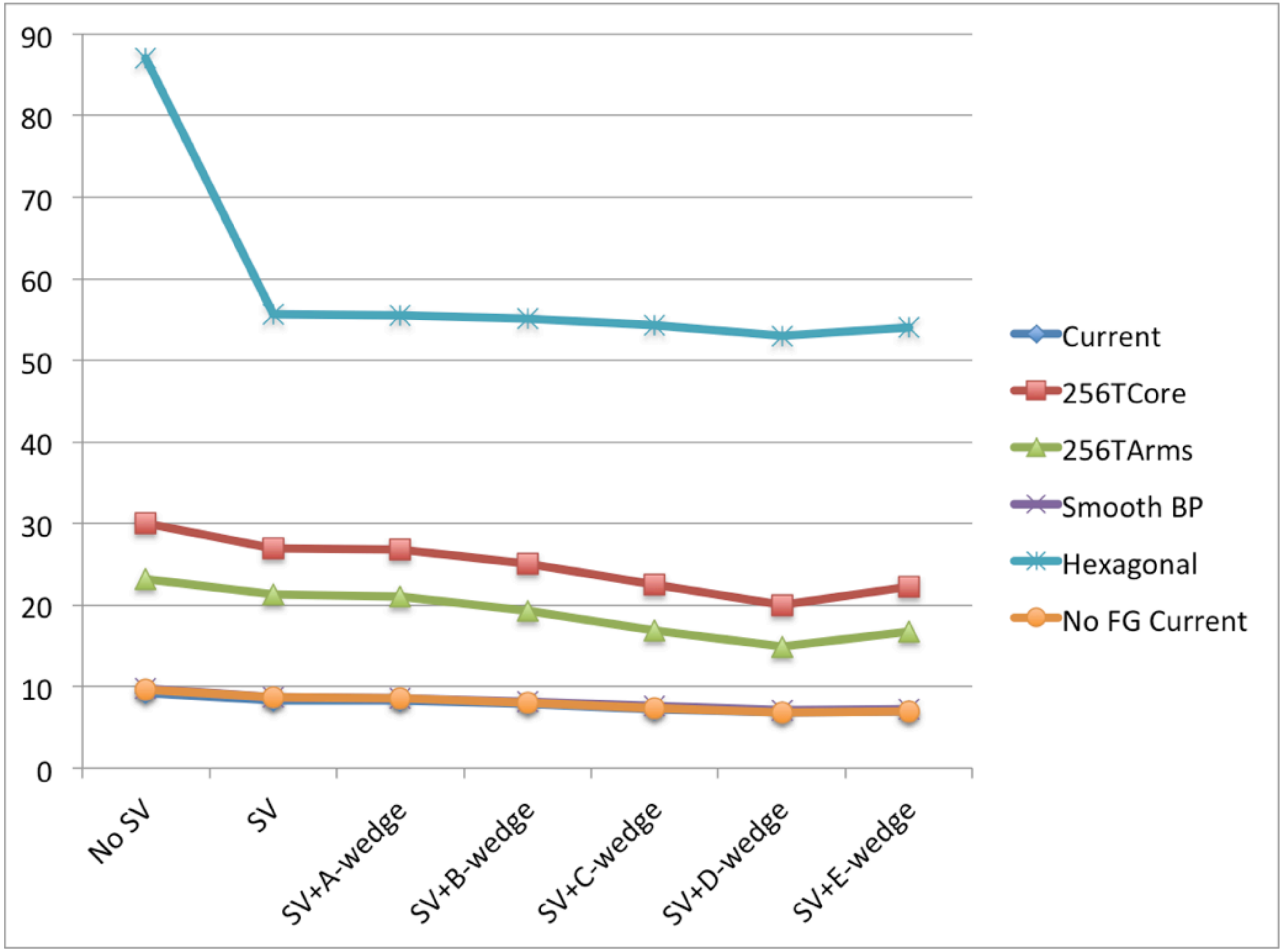
1D Power Spectrum - exclude $k_{||} > 0.1$ & $k_{||} > 3k_{\perp}$





SNRs: slope α

SNR



Conclusions

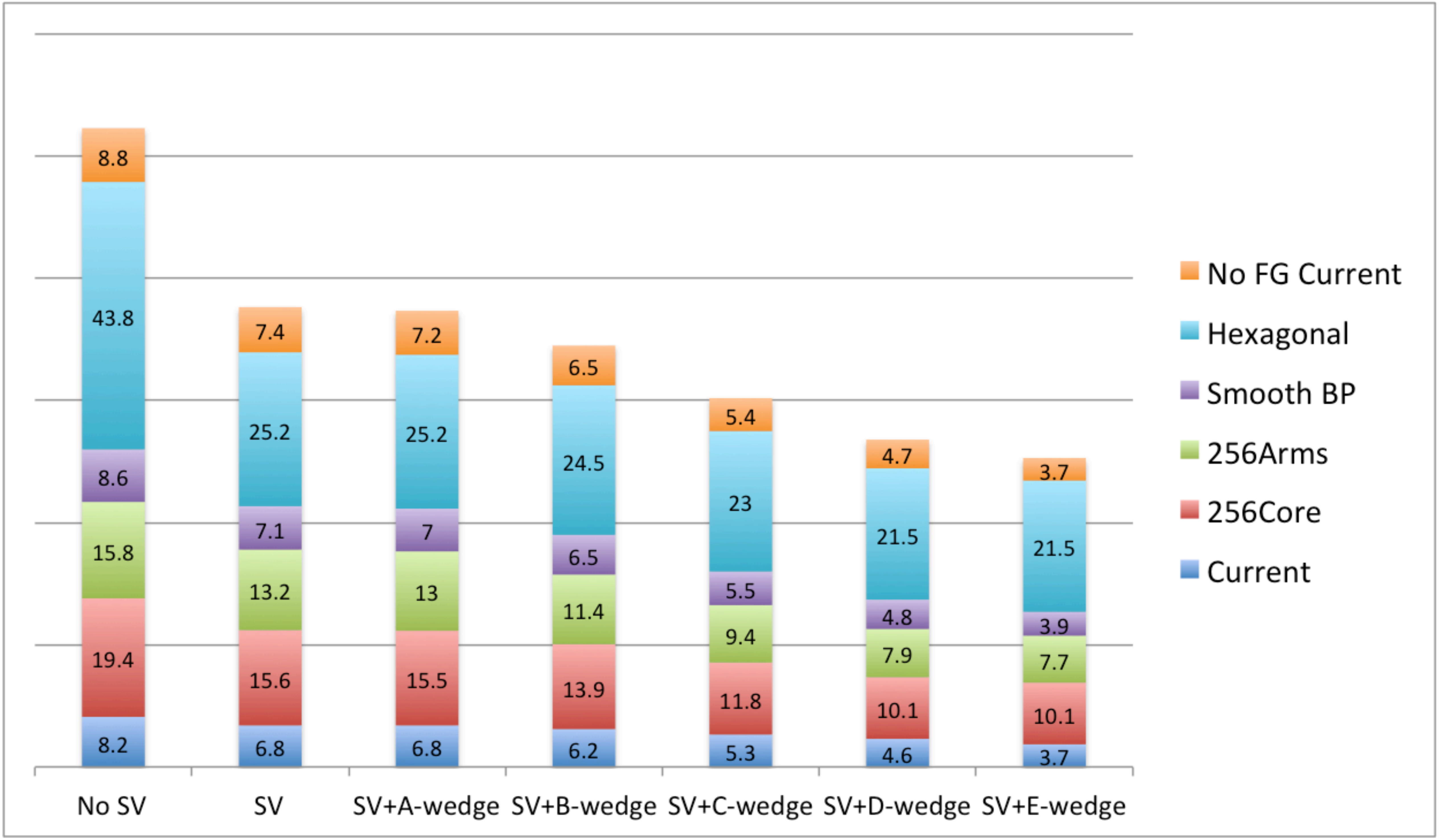
- **Building sensitivity at the right angular scales is key**
  - Enforced coherence (redundancy) is useful, but not required for *sensitivity* (may be extremely useful for calibration and removing systematics) ---> rotation synthesis rotates quasi-redundant baselines into redundancy; useful for physical tile placement
  - A smooth bandpass does not impact sensitivity, but is more crucial for instrument calibration (systematics)
  - Foreground leakage into EoR Window is still likely to impact, requiring long baselines to perform the best calibration

- **MWA can perform detection experiment, and basic estimation. MWA+ can perform extended estimation (possibly 2D estimation)**

SNRs: slope α

9.6
87.1
9.7
23.2
30
9.2
8.7
55.6
8.7
21.3
26.9
8.3
8.5
55.5
8.6
21
26.8
8.3
8
55.1
8.2
19.3
25.1
7.9
7.3
54.3
7.6
16.9
22.5
7.2
6.8
53
7.1
14.9
20
6.8
6.9
54
7.2
16.7
22.2
6.9
No SV
SV
SV+A-wedge
SV+B-wedge
SV+C-wedge
SV+D-wedge
SV+E-wedge
No FG Current
Hexagonal
Smooth BP
256Arms
256Core
Current

SNRs: amplitude $\Delta^2(k_p)$



1D Power Spectra - *no* sample variance

$$\Delta P = \left(\sum_k \left(\mathcal{H}^\dagger C^{-1} \mathcal{H} \right)^2 \right)^{-1/2} \simeq \frac{1}{\sqrt{M_k}} (\boldsymbol{N}(k) + \boldsymbol{C}_{\mathrm{FG}}(k))$$