Halo-independent tests of dark matter direct detection signals

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Outline

1 Introduction to halo-independent (HI) methods

2 How to test HI direct detection (DD) signals with LHC, relic abundance, indirect detection, local energy density...
$\rightarrow$ Extension to annual modulation signals

3 How to test HI DD rates with neutrinos from the Sun
$\rightarrow$ Extension to inelastic DM

4 Inelastic self-interactions to solve small-scale structure problems

5 Summary and conclusions

Introduction to halo-independent methods

Motivation: derive new HI frameworks for DD

Uncertainties on $f(v)$, ρ_χ are crucial for interpretation of DM DD signals:

$$\mathcal{R}(E_R, t) = A_{\text{eff}}^2 F_A^2(E_R) \tilde{\eta}(v_m, t), \qquad \tilde{\eta}(v_m, t) \equiv \mathcal{C} \int_{v_m}^{\infty} dv v \tilde{f}_{\text{det}}(v, t),$$

where $A_{\text{eff}}^2 = (Z + f_n/f_p\,(A - Z))^2$, $\delta = m_\chi^* - m_\chi$ and:

$$v_m = \left| \sqrt{\frac{m_A E_R}{2\mu_{\chi A}^2}} + \frac{\delta}{\sqrt{2\, m_A\, E_R}} \right|, \qquad \mathcal{C} \equiv \frac{\rho_\chi \sigma_{\text{SI/SD}}}{2 m_\chi \mu_{\chi p}^2}.$$

- Halo-independent (HI) tests of DD signals independently of $f(v)$:

  $\longrightarrow$ For fixed m_χ, compare $\tilde{\eta}(v_m, t)$, which is detector-independent, in same v_m range [Fox, Gondolo, Bozorgnia, Feldstein, JHG...].

- We propose two new HI methods to compare a positive DD signal:

1. With limits from the LHC, ID, the thermal DM hypothesis, ρ_χ...
2. With the neutrino rate from the Sun, Γ_{Sun}.

Testing direct detection rates with LHC limits, indirect detection, relic abundance, ρ_χ...

JCAP (2015) 1508 08, 039 , arXiv [hep-ph]: 1505.05710

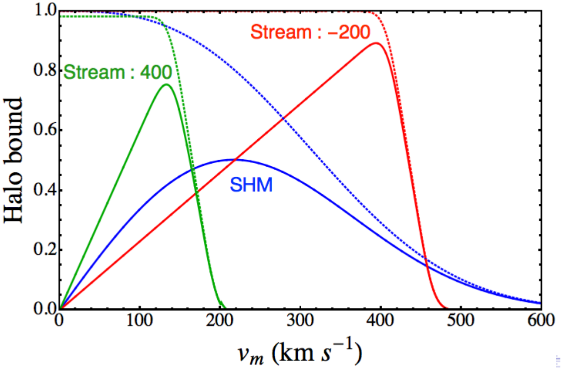
JCAP (2015) 1509 09, 012 , arXiv [hep-ph]: 1506.03503

Lower bound on the halo function

[Feldstein, Kavanagh, Blennow...]

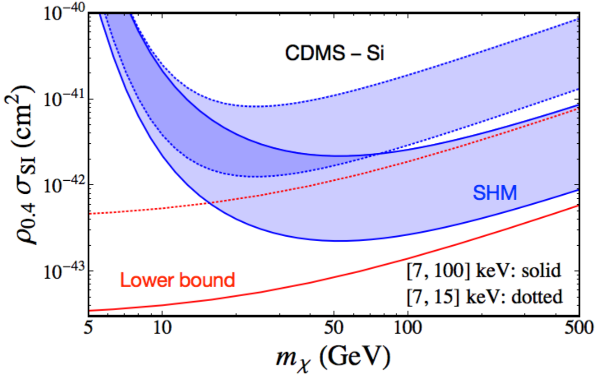
$$1 \equiv \int_0^\infty d^3v \, f_{\rm det}(\vec{v}) \equiv \int_0^\infty \overline{\eta}(v) dv \quad \geq \overline{\eta}(v_1)\, v_1 + \int_{v_1}^{v_2} dv\, \overline{\eta}(v) \quad B_1 \,(\text{dotted})$$

$$\geq \overline{\eta}(v_1)\, v_1 \qquad B_2\,(\text{solid})$$



Lower bound on $\rho_\chi \sigma_{\rm SI/SD}$ from # of events (CDMS-Si)

$$B_2 \longrightarrow \rho_\chi \sigma_{\rm SI} \geq \frac{2\, m_\chi\, \mu_p^2}{MT\, \langle 1/v_m^A \rangle_{E_1}^{E_2}} N_{[E_1,E_2]} \qquad \text{“Events bound”}$$



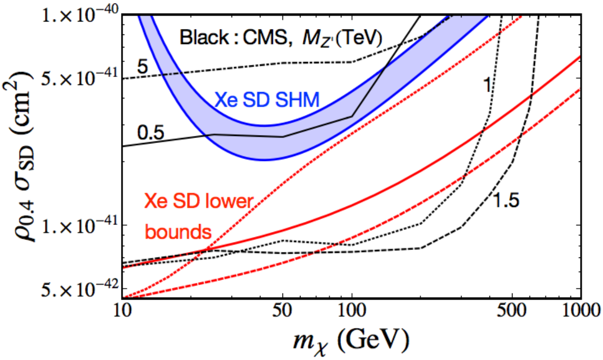
$\longrightarrow \sigma_{\rm SI} \lesssim 3 \cdot 10^{-43}\,{\rm cm}^2$ are disfavoured.

Lower bound on $\rho_\chi \sigma_{\rm SI/SD}$ from a DD spectrum (Xe SD)

$$B_1 \to \rho_\chi \sigma_{\rm SI} \geq \frac{2 m_\chi \mu_{\chi p}^2}{A^2} \left(v_1 \frac{\mathcal{R}(E_1)}{F_A^2(E_1)} + \int_{v_1}^{v_2} dv \frac{\mathcal{R}(E_R)}{F_A^2(E_R)} \right) \text{ “Spectrum bound”}$$

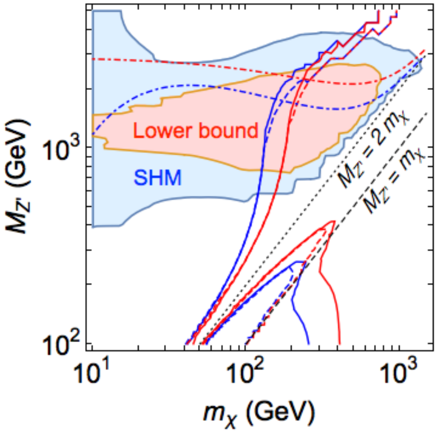
Simplified model: Majorana fermion χ, equal couplings to u, d, s, c:

$$\mathcal{L}_{\rm int} = g_\chi \bar{\chi} \gamma_\mu \gamma^5 \chi Z'^\mu + g_q \bar{q} \gamma_\mu \gamma^5 q Z'^\mu$$



Constraints from LHC and relic abundance ($\rho_\chi = 0.4$)

- Shaded area: LHC limits. $\Gamma_{Z'} > M_{Z'}/2$ (dotted-dashed).
- Ω_χ bound also valid for multi-component if DD given by one species, and $\rho_\chi \propto \Omega_\chi$ (CDM). Conservative: more channels make it stronger.

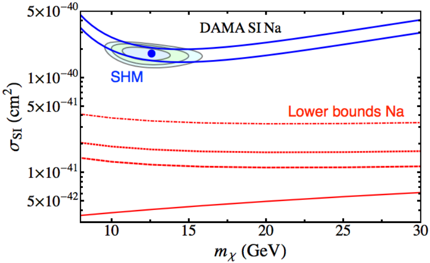


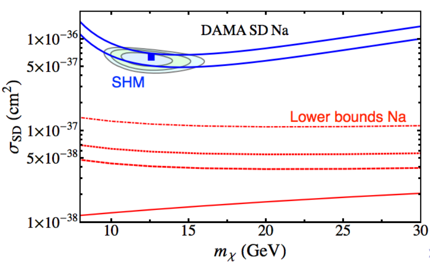
1. Only time-dep.in $v_e(t)$, f constant, t_0 free:

$$\rho_\chi \sigma_{\rm SI} \geq \frac{2\, m_\chi\, \mu_p^2}{A^2} \frac{1}{v_e} \left(\frac{2}{v_1} + \frac{1}{v_2} \right)^{-1} \int_{v_1}^{v_2} dv \frac{\mathcal{M}(v)}{F_A^2(E_R)} \qquad \text{"General"}$$

2. Fixed DM flow, t_0 constant. $\sin\alpha = 0.5, t_0 =$June 2nd (SHM).

$$\rho_\chi \sigma_{\rm SI} \geq \frac{2\, m_\chi\, \mu_p^2}{A^2} \left(\frac{1}{v_1} \right)^{-1} \frac{1}{\sin\alpha\, v_e} \int_{v_1}^{v_2} dv \frac{\mathcal{M}(v)}{F_A^2(E_R)} \qquad \text{"Symmetric"}$$





Testing direct detection signals with neutrinos from the Sun

JCAP 1505 (2015) 05, 036 , arXiv [hep-ph]: 1502.03342

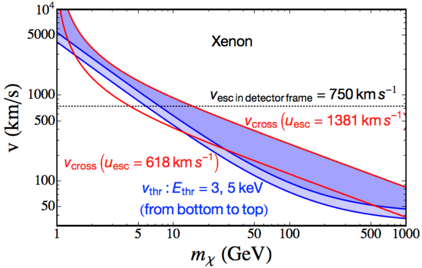
JCAP 1604 (2016) no.04, 004 , arXiv [hep-ph]: 1512.03317

Overlap in velocity (H, SD) and assumptions of the bound

[Gould, Edsjo, Kavanagh, Blennow...]

Overlap in $v_{\rm thr} < v < v^A_{\rm cross}(r)$:

- DD is sensitive to $v > v_{\rm thr}$.
- Capture for $v < v^A_{\rm cross}(r)$.



We can derive a lower bound on the solar capture, assuming:

1. We use that $v_e \approx 29\,{\rm km/s} \ll v_m$ so that:

$$\tilde{f}_{\rm det}(v) = \tilde{f}_{\rm Sun}(v + v_e) \approx \tilde{f}_{\rm Sun}(v) \equiv \tilde{f}(v)\,.$$

2. $f(v)$, ρ_χ constant so they are equal for the capture and for DD.

A lower bound on the capture rate in the Sun

$$C_{\rm Sun} = 4\pi\,\mathcal{C} \sum_A A_{\rm eff}^2 \int_0^{R_{\rm S}} dr r^2 \rho_A(r) \int_0^{v_{\rm cross}^A} dv \tilde{f}(v)\, v\, \mathcal{F}_A(v,r)$$

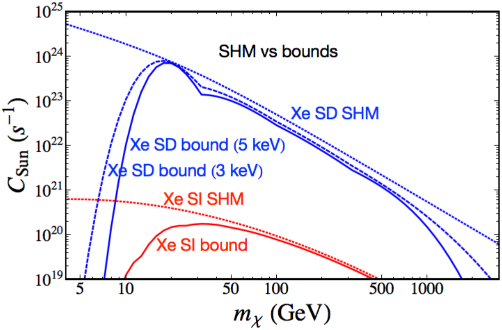
$$\geq 4\pi\,\mathcal{C} \sum_A A_{\rm eff}^2 \int_0^{R_{\rm S}} dr r^2 \rho_A(r) \int_{v_{\rm thr}}^{v_{\rm cross}^A} dv\, \tilde{f}(v)\, v\, \mathcal{F}_A(v,r).$$

From a perfectly measured DD spectrum one can extract:

$$\mathcal{C}\tilde{f}(v) = -\frac{1}{v}\frac{d\tilde{\eta}(v)}{dv} = -\frac{1}{v A_{\rm eff}^2}\frac{d}{dv}\left(\frac{\mathcal{R}(E_R)}{F_A^2(E_R)}\right)$$

- The bound on $C_{\rm Sun}$ can be expressed in terms of DD quantities.
- It is independent of $f(v)$, $v_{\rm esc}$, $\sigma_{\rm SI/SD}$ and ρ_χ.

Comparison with the SHM



Our bounds are strongest:

- For Xe in $20 \lesssim m_\chi \lesssim 1000$ GeV (SD) and $m_\chi \gtrsim 50$ GeV (SI).

Inelastic scattering: $\chi N \to \chi^* N$ [Smith, Harnik]

[Blennow, Clementz, JHG]

Using the shape test one can check compatibility with inelastic

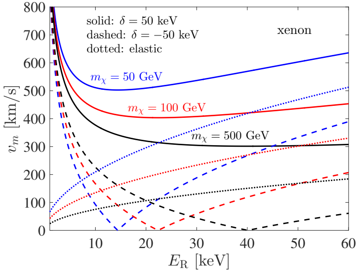
[Bozorgnia, Schwetz, Zupan, JHG]

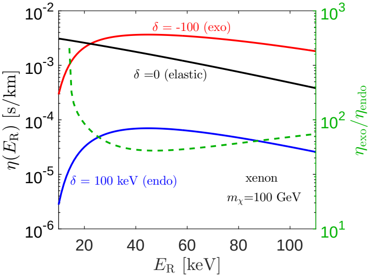
Minimum velocity:

$$v_m = \left| \sqrt{\frac{m_A E_R}{2\mu_{\chi A}^2}} + \frac{\delta}{\sqrt{2\, m_A\, E_R}} \right|$$

Maximum of $\eta(E_R)$ at:

$$\longrightarrow E_{\rm min}^{\rm obs} = \frac{\mu}{m_{\rm A}} |\delta| \,.$$





Recipe after observing two inelastic signals

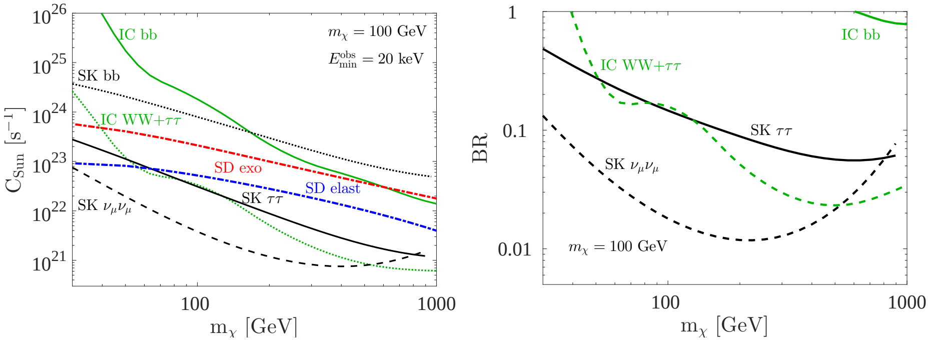
1. Check if both signals are compatible with inelastic (the shape test).
2. If so, extract mass and the absolute value of the splitting:

$$m_\chi = \frac{m_1 E_1^{\rm obs} - m_2 E_2^{\rm obs}}{E_2^{\rm obs} - E_1^{\rm obs}} \,,$$

$$|\delta| = \frac{E_1^{\rm obs} E_2^{\rm obs} (m_1 - m_2)}{m_1 E_1^{\rm obs} - m_2 E_2^{\rm obs}} \,.$$

3. Go to v space and check whether endo and/or exo fit better.
4. Extract the couplings to n and p with the ratio of the signals in the same v region: $A^2_{\exp 2}/A^2_{\exp 1}$.
5. Apply our lower bounds on the capture and derive bounds on BR into different annihilation channels if there is no neutrino signal.

Inelastic capture and lower bounds for $E_{\rm min}^{\rm xenon} = 20$ keV.



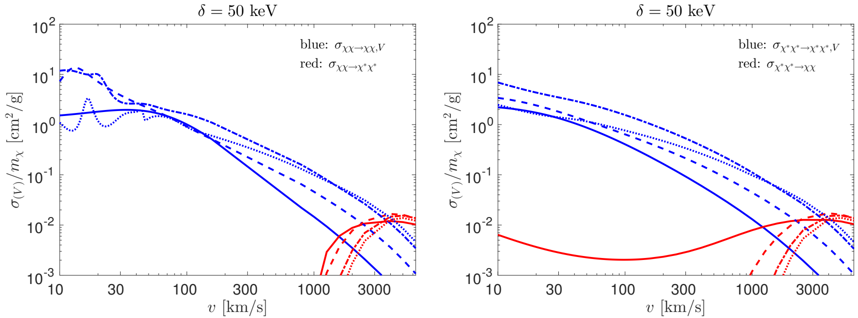
Self-interacting inelastic dark matter: A viable solution to the small scale structure problems

arXiv [hep-ph]: 1612.06681

Inelastic self-interactions for small-scale structure problems

[Blennow, Clementz, JHG] ([also Slatyer, Weiner, Kaplinghat, Zurek, Ershov, Zhang...])

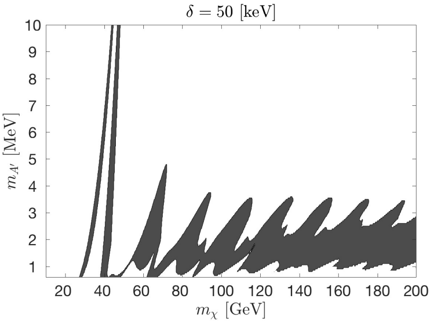
- Light mediators generate large self interactions capable of solving small-scale structure problems, but strong DD limits on the mixing with SM are usually incompatible with them decaying before BBN.
- Solution: Inelastic DM with large self. ints. $\chi\chi \to \chi\chi$, $\chi^*\chi^* \to \chi^*\chi^*$, $\chi\chi \to \chi^*\chi^*$, $\chi^*\chi^* \to \chi\chi$, evading DD for $\delta \gtrsim 50$ keV IF $n_{\chi^*} \ll n_\chi$.
- As interactions are long-range, need to solve numerically the Schr.eq.

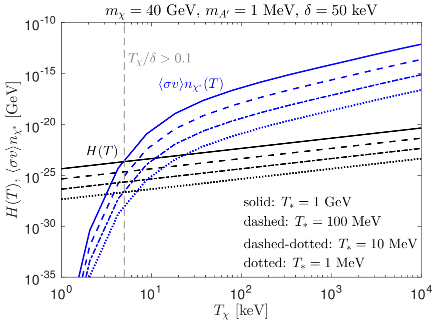


Viable parameter space and abundance of excited state

[Blennow, Clements, JHG, 1612.06681] ([also Finkbeiner, Batell])

- Even if χ^* is stable, the large self-interactions $\chi^*\chi^* \to \chi\chi$ in the Early Universe suppress n_{χ^*} if they are still efficient at $T_\chi^{\rm dd} < \delta$.
- We get $n_{\chi^*}/n_\chi \lesssim \mathcal{O}(0.1)$, and typically less, suppressing DD limits!





Summary and conclusions

Summary and conclusions

New HI methods to compare positive DD signals with other DM searches:

1. We have derived a HI lower bound on $\rho_\chi \, \sigma_{\rm SI/SD}$, which allows to bound models with limits from LHC, Ω_χ, ID or ρ_χ.
   - We have extended it to annual modulations, using an expansion on the Earth's velocity, v_e. Applied to DAMA.

2. We derived a lower bound on the capture rate in the Sun that is independent of $f(v)$, $v_{\rm esc}$, $\sigma_{\rm SI/SD}$ and ρ_χ, with which, assuming equilibrium, one can bound models with neutrino rate limits.
   - It is strong for SD and channels to $\nu\nu$, $\tau\tau$ and $m_\chi \gtrsim 100$ GeV.
   - We extended it to inelastic scattering, strong bounds for exth. SD.
   - From 2 signals one gets HI m_χ, $|\delta|$, f_n/f_p and the endo/exo nature.

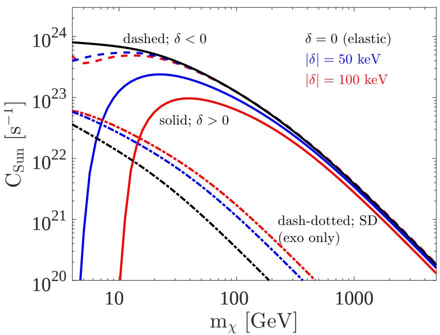
- Inelastic self-interactions are a viable solution to the small scale structure problems, suppressing DD (as typically $n_{\chi^*}/n_\chi \ll 1$) for large enough δ.

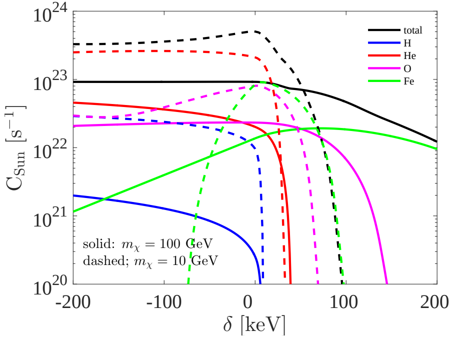
Complementarity of the signals in DD and in the Sun

DD	$\Gamma_{\rm Sun}$	**Lesson**
No	No	Keep trying... Axions? Sterile neutrinos? Eventually, does DM interact non-gravitationally?
No	Yes	There is no halo-independent lower bound on $\mathcal{R}_{\rm DD}$ from a ν signal Dark disk? [Bruch, Choi...] Self-interactions? [Zentner...] SD dominated by protons? Inelastic (endo/exo)? [Nussinov, Menon, Shu, JHG...]
Yes	No	Halo-independent lower bound on capture. $\rightarrow$ Upper bounds on branching ratios [Blennow et al]. SD dominated by neutrons? Asymmetric DM with suppressed Γ_ν? [Kaplan, Nussinov...] Inelastic (endo/exo)? [Nussinov, Menon, Shu, JHG...]
Yes	Yes	Check if the lower bounds here derived are fulfilled. If so, extract DM properties by a fit [Arina, Serpico, Kavanagh...].

Back-up slides

Inelastic capture and lower bounds for $E_{\min}^{\text{xenon}} = 20$ keV.





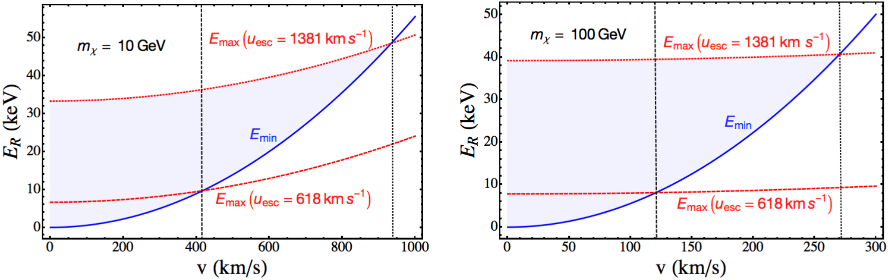
Capture: there is a maximum velocity v_{cross} (shown H, SD)

For the DM to be captured there is a minimal and maximal E_R:

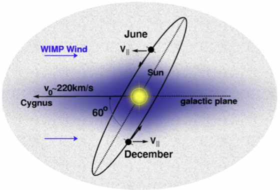
$$E_{\min} = \frac{m_\chi}{2} v^2 \,, \qquad E_{\max} = \frac{2\mu_{\chi A}^2}{m_A}(v^2 + u_{\text{esc}}^2(r)) \,.$$

These define the maximum velocity to be trapped:

$$v_{\text{cross}}^A(r) = \frac{\sqrt{4 m_A m_\chi}}{|m_\chi - m_A|} u_{\text{esc}}(r)$$



Halo-independent bounds on annual modulation in DD



Annual modulation [Freese et al]:

Depending on the time of the year, we should *receive* more or less DM flux in our detectors.

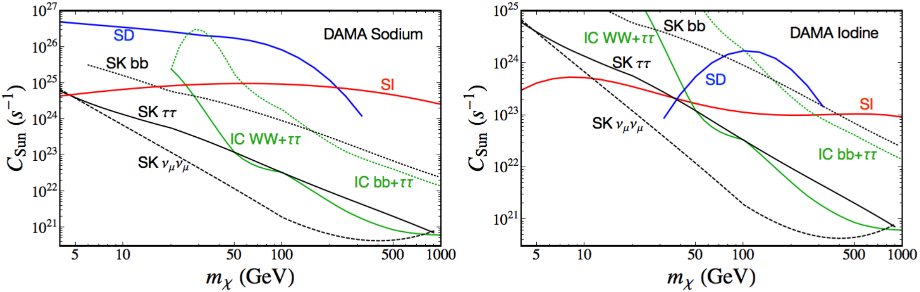
- The annual modulation $A_\eta(v_m)$ can be constrained in terms of the constant rate $\bar{\eta}(v_m)$ (almost) halo-independently [JHG, Schwetz, Zupan], by expanding $\eta(v,t)$ in $v_e/v \ll 1$, with $v_e \simeq 30\,\text{km/s}$.
- If there is a preferred direction in the DM velocity:

$$A_\eta(v_m) \leq -v_e \sin\alpha_{\text{halo}} \frac{d\bar{\eta}}{dv_m}.$$

- Therefore there is also a lower bound on C_{Sun} from $A_\eta(v_m)$:

$$C_{\text{Sun}} \geq 4\pi \sum_A A^2 \int_0^{R_{\text{Sun}}} dr r^2 \rho_A(r) \int_{v_{\text{thr}}}^{v^A_{\text{cross}}} dv \, \frac{\tilde{A}_\eta(v)}{\sin\alpha_{\text{halo}}\, v_e} \mathcal{F}_A(v).$$

DAMA (already strongly disfavored HI by DD) ($f_n = f_p$)



DAMA in strong tension:

- Na: for SI annihilation into $\nu\nu$, $\tau\tau$ (bb) are strongly constrained for $m_\chi \gtrsim 5\,(15)$ GeV, while SD is excluded for all channels.
- I: for SI, strong bounds for $m_\chi \gtrsim 10, 35$ GeV for $\nu\nu$, $\tau\tau$. For SD $m_\chi \gtrsim 40\,(80)$ GeV excluded for $\nu\nu$, $\tau\tau$ (bb).

Equilibrium between capture and annihilations in the Sun

- DM obeys ($A_{\rm Sun}$ annihilation rate, no evaporation for $m_\chi \gtrsim 3.5$ GeV):

$$\frac{dN}{dt} = C_{\rm Sun} - A_{\rm Sun} N^2 \,.$$

- Equilibrium ($dN/dt = 0$) occurs for $t_{\rm eq} \ll t_{\rm Sun} \sim 4.5\,{\rm Gyr}$, where:

$$t_{\rm eq} = \frac{1}{\sqrt{C_{\rm Sun}\, A_{\rm Sun}}} \approx$$

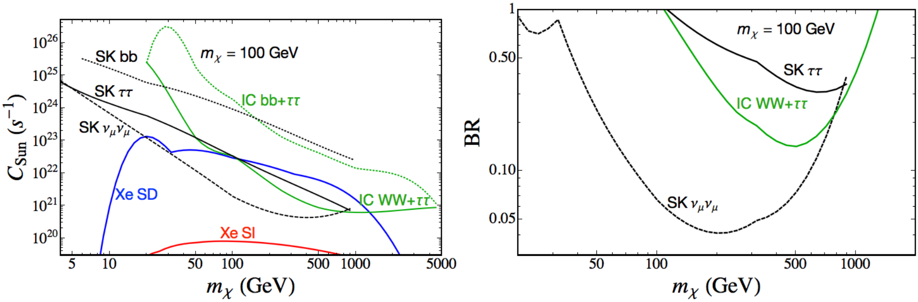
$$\approx 0.5\,{\rm Gyr} \left(\frac{10^{21}\,{\rm s}^{-1}}{C_{\rm Sun}}\right)^{1/2} \left(\frac{3\cdot 10^{-26}\,{\rm cm}^3\,{\rm s}^{-1}}{\langle\sigma v\rangle_{\rm ann}}\right)^{1/2} \left(\frac{100\,{\rm GeV}}{m_\chi}\right)^{3/4},$$

where $\langle\sigma v\rangle_{\rm ann}$ is the thermally-averaged annihilation cross section.

- For $C_{\rm Sun} \lesssim 10^{21}\,{\rm s}^{-1}$, no equilibrium reached if $\langle\sigma v\rangle_{\rm ann} < \langle\sigma v\rangle_{\rm fo}$.
- If it is reached $\Gamma_{\rm Sun}$ given solely in terms of $C_{\rm Sun}$:

$$\Gamma_{\rm Sun}^{\rm eq} = \frac{N^2}{2} A_{\rm Sun} = \frac{C_{\rm Sun}}{2}.$$

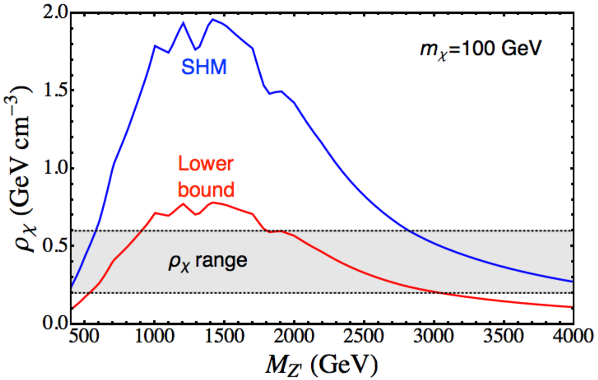
$C_{\rm Sun}$ and upper bounds on BR for Xe mock data ($f_n = f_p$)



Results:

- For xenon, for SD, annihilations into ν would be constrained to BR at the few % level. $\tau\tau, WW$ at the 10% level.
- Strong dependence on m_χ. Stronger bounds at the *wrong* m_χ.

Lower bound on ρ_χ for Xe SD from LHC limits

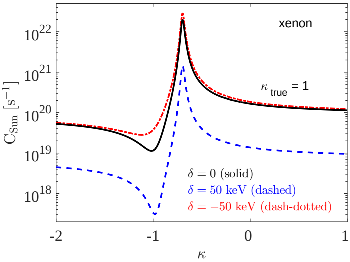


Unknown couplings $f_n \neq f_p$, and uncertainties in SD FF

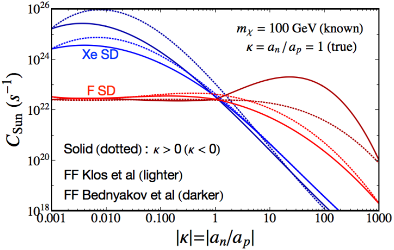
The extracted $f(v)$ depends on couplings to n and p, and FF:

$$\mathcal{C}\tilde{f}_{\rm extr}(v) = \mathcal{C}\tilde{f}(v)\frac{A^2_{\rm true}\,F^2_{\rm true}(E_R)}{A^2_{\rm wrong}\,F^2_{\rm wrong}(E_R)} - \frac{\tilde{\eta}(v)}{v}\frac{A^2_{\rm true}}{A^2_{\rm wrong}}\frac{d}{dv}\left(\frac{F^2_{\rm true}(E_R)}{F^2_{\rm wrong}(E_R)}\right).$$

Isospin violation (IV), $\kappa \equiv f_n/f_p$:



SD Xe (n) & F (p), $\kappa \equiv a_n/a_p$:



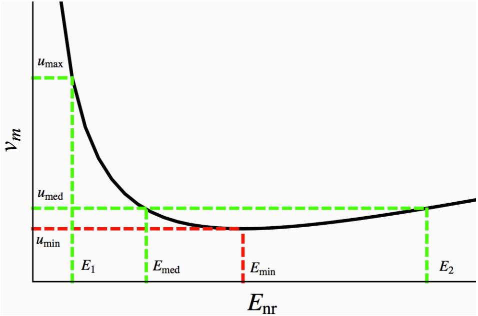
Left) $\kappa_{\rm true} = 1$, $m_\chi = 100$ GeV, SI with 10^{-45} cm^2 and $|\delta| = 50$ keV.
Right) Lower bound for SD interactions for F (p) and Xe (n).

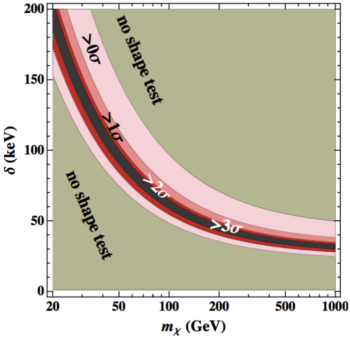
Inelastic scattering. Example of the *shape test*

Bozorgnia, Schwetz, Zupan, JHG.

$$v_m = \sqrt{\frac{m_A E_r}{2\mu_{\chi A}^2}} + \frac{\delta}{\sqrt{2 m_A E_r}}$$

$$\frac{\left| \int_{E_{min}}^{E_{med}} - \int_{E_{min}}^{E_2} \right|}{\sqrt{\sigma_a^2 + \sigma_b^2}}$$ for DAMA (I):





A lower bound on the capture

$$C_{\text{Sun}} \geq 4\pi \sum_A A^2 \int_0^{R_S} dr r^2 \rho_A(r) \int_{v_{\text{thr}}}^{v^A_{\text{cross}}} dv \left(-\frac{d\tilde{\eta}(v)}{dv}\right) \mathcal{F}_A(v,r)$$

$$= 4\pi \sum_A A^2 \int_0^{R_S} dr r^2 \rho_A(r) \left[\tilde{\eta}_{\text{thr}} \mathcal{F}_A(v_{\text{thr}},r) + \int_{v_{\text{thr}}}^{v^A_{\text{cross}}} dv\, \tilde{\eta}(v)\, \mathcal{F}'_A(v,r)\right],$$

where in the last line we integrated by parts, with $\mathcal{F}_A(v^A_{\text{cross}}, r) = 0$.

Features:

- Either the derivative or the function $\tilde{\eta}(v)$, including its value at the threshold, have to be determined from DD.
- The bound is independent of the DM velocity distribution, the galactic escape velocity, the scattering cross section and the local DM density.

Numerical examples

We simulate mock data motivated by future experiments:

- Xenon, with $\sigma_{\rm SI} = 10^{-45}\,{\rm cm}^2$ and $\sigma_{\rm SD} = 2 \cdot 10^{-40}\,{\rm cm}^2$. Assuming $m_\chi = 100$ GeV, for an exposure of 1 ton yr, about 154 (267) events in the range $5 - 45$ keV for SI (SD) are predicted.
- Germanium, with $E_{\rm thr} = 1$ keV, focusing on low DM masses. Assuming $m_\chi = 6$ GeV and $\sigma_{\rm SI} = 5 \cdot 10^{-42}\,{\rm cm}^2$ and $\sigma_{\rm SD} = 2 \cdot 10^{-40}\,{\rm cm}^2$, 1.5×10^4 (2–3) events for SI (SD) predicted in the range 1–10 keV for an exposure of 100 kg yr with energy resolution of 30%.

Capture rate in the Sun [Press, Griest, Gould, Bergstrom, Edsjo, Blennow...]

- The DM velocity inside the gravitational potential of the Sun $w^2 = v^2 + u_{\rm esc}^2(r)$, with $u_{\rm esc}^2(r)$ the escape velocity from the Sun.
- The capture rete is given by (notice that $wdw = vdv$)

$$C_{\rm Sun} = 4\pi \frac{\rho_\chi}{m_\chi} \sum_A \int_0^{R_{\rm S}} dr\, r^2 \int_0^\infty dv\, \tilde{f}(v)\, v\, w\, \Omega_A(w,r)\,,$$

with

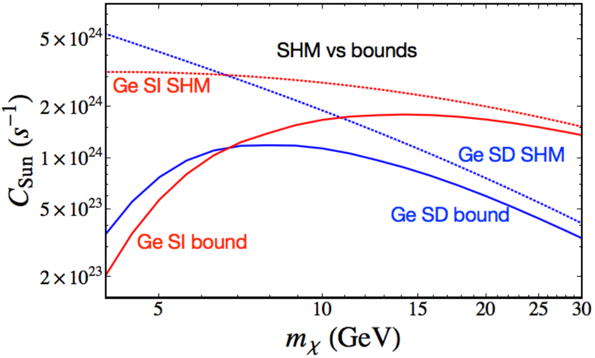
$$\Omega_A(w,r) = w\, \frac{\rho_A}{m_A} \int_{E_{\rm min}(w)}^{E_{\rm max}(w)} dE_R \frac{d\sigma_A}{dE_R}(w)\,,$$

where E_R is the nuclear recoil energy.
$\mathcal{F}_A(v,r) \equiv \int_{E_{\rm min}(v)}^{E_{\rm max}(v)} F_A^2(E_R) dE_R$.

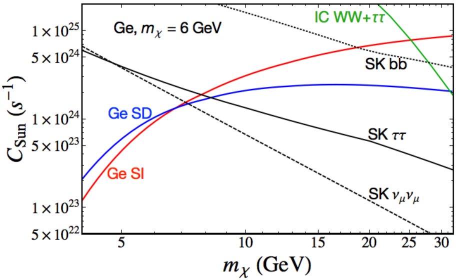
If equilibrium between capture and annihilation is reached:

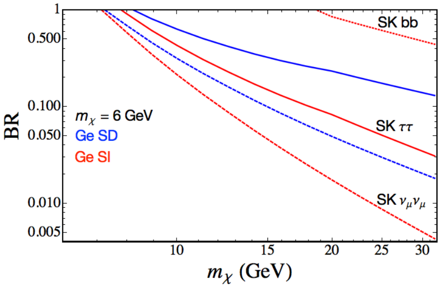
$$\Gamma_{\rm Sun} = \frac{1}{2} C_{\rm Sun}$$

C_{Sun} for Ge



Results for Ge

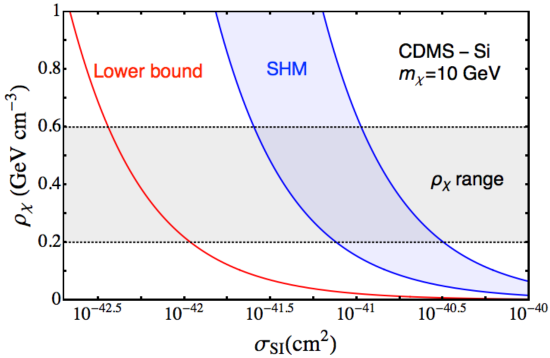




Results:

- For xenon, for SD, annihilations into ν would be constrained to BR at the few % level. $\tau\tau, WW$ at the 10% level.
- For germanium, for both SD and SI direct annihilations into ν would be constrained to BR at the few % level. $\tau\tau$ are at *wrong* m_χ.
- Strong dependence on m_χ. Stronger bounds at the *wrong* m_χ.

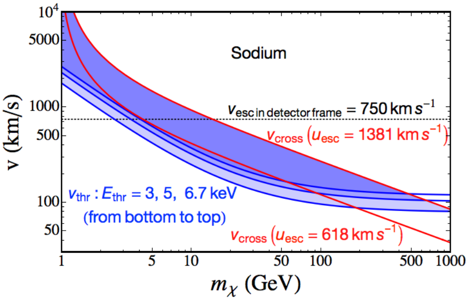
CDMS

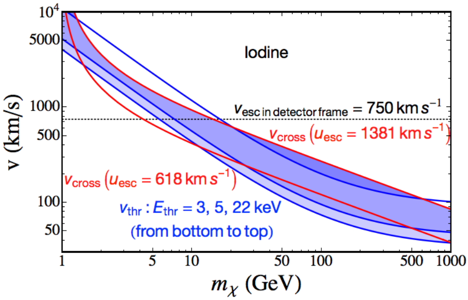


CDMS – Si
m_χ=10 GeV
Lower bound
SHM
ρ_χ range
ρ_χ (GeV cm^{-3})
1
0.8
0.6
0.4
0.2
0
$10^{-42.5}$
10^{-42}
$10^{-41.5}$
10^{-41}
$10^{-40.5}$
10^{-40}
σ_{SI}(cm^2)

DAMA results

- Na dominates for DM masses $m_\chi \lesssim 20$ GeV.
- Iodine is relevant for larger DM masses.
- Small overlap for iodine for hydrogen (SD).





Expansion of $\eta(v_m, t)$ in v_e/v

$v_{esc} \gg \langle v \rangle > v_m \gg v_e$, so we can expand $\eta(v_m, t)$ to first order in v_e:

$$\eta(v_m, t) = \int_{v_m} d^3v \, \frac{f_{\rm det}(\vec{v})}{v} = \int_{v_m} d^3v \, \frac{f_{\rm Sun}(\vec{v} + \vec{v_e}(t))}{v} =$$

$$= \int_{v_m} d^3v \, \frac{f_{\rm Sun}(\vec{v})}{v} + \int d^3v \, f_{\rm Sun}(\vec{v}) \, \frac{\vec{v}\cdot\vec{v_e}(t)}{v^3} \left[\Theta(v - v_m) - \delta(v - v_m)\, v_m\right] \equiv$$

$$\equiv \overline{\eta}(v_m) + A_\eta(v_m) \cos 2\pi(t - t_0).$$

- $\overline{\eta}(v_m)$ is constant, A_η is modulated, with observed rates:

$$\overline{R} \equiv \mathcal{C}F^2(E_r)\overline{\eta}(v_m) \;\; \text{and} \;\; A_R \equiv \mathcal{C}F^2(E_r)A_\eta$$

The general bound on the annual modulation

1. Halo "smooth" on $\lesssim v_e \sim 30\,\mathrm{km/s}$.
2. Only time dependence in $v_e(t)$, not in f_{Sun} (no change on months).

$$\int_{v_{m1}}^{v_{m2}} dv_m\, A_\eta(v_m) \leqslant v_e \left[\overline{\eta}(v_{m1}) + \int_{v_{m1}} dv\, \frac{\overline{\eta}(v)}{v} \right]$$

3. If there is a constant $\hat{v}_{HALO}$ governing the modulation:

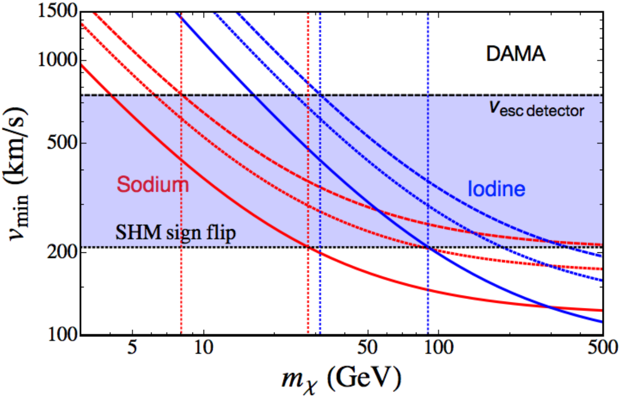
$$\int_{v_{m1}}^{v_{m2}} dv_m\, A_\eta(v_m) \leqslant \sin\alpha\, v_e\, \overline{\eta}(v_{m1})$$

where:

- in general $\sin\alpha$ can be set to 1.
- $\sin\alpha = 0.5$ when $\hat{v}_{HALO} \propto \hat{v}_{SUN}$ (isotropic, SHM, DD...). Then $t_0 =$ June 1st.

DAMA signal in the SHM

- Sodium (red) and iodine (blue) for $E_R = 2,\ 4\,,6\,\mathrm{keVee}$ (from bottom to top as solid, dotted and dashed curves).
- Dotted black below which $\mathcal{M} < 0$, and as dashed black the typical escape velocity in the detector rest frame.
- SHM: $8 \lesssim m_\chi\,(\mathrm{GeV}) \lesssim 30$ ($30 \lesssim m_\chi\,(\mathrm{GeV}) \lesssim 90$) for Na (I)

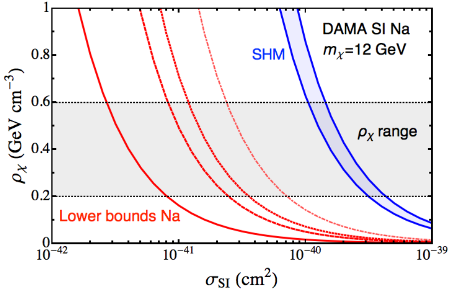


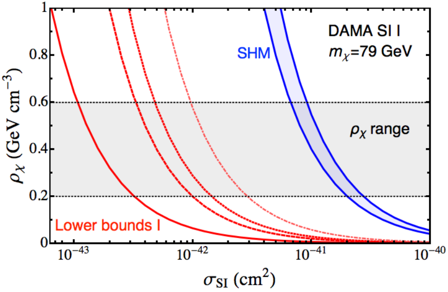
DAMA fit to the SHM

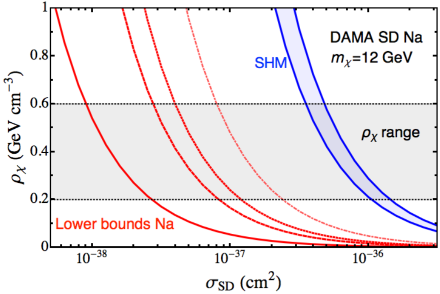
$v_{\rm esc} = 550$ km/s, $\rho_\chi = 0.4\,{\rm GeV\,cm^{-3}}$, $q_{\rm Na} = 0.3$ and $q_{\rm I} = 0.09$. Equal couplings to protons and neutrons

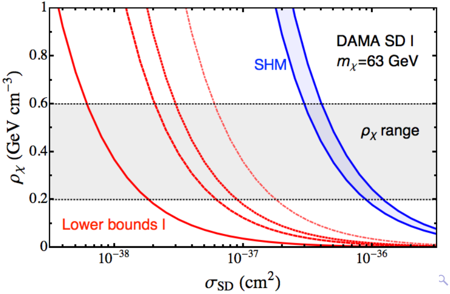
	I			**Na**		
	m_χ (GeV)	$\sigma_{\rm SI/SD}$ (cm^2)	$\chi^2_{\rm min}$/dof	m_χ (GeV)	$\sigma_{\rm SI/SD}$ (cm^2)	$\chi^2_{\rm min}$/dof
SI	79.4	$1.1 \cdot 10^{-41}$	7.7/6	12.6	$1.8 \cdot 10^{-40}$	8.3/6
SD	63.1	$5.0 \cdot 10^{-37}$	7.9/6	12.6	$6.3 \cdot 10^{-37}$	8.7/6

Local energy density



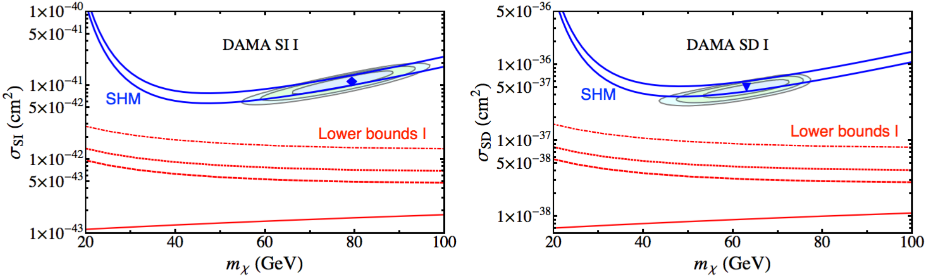






Example: DAMA (already strongly disfavoured HI by DD)

Blue: SHM. Red: from bottom "Spectrum" solid, "General" dashed and "Symmetric" dotted ($\sin\alpha = 1$), dotted-dashed ($\sin\alpha = 0.5$).



Multi-target bounds

Bounds (in black) for DAMA modulation for SI (top) and SD (bottom)